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一类具有阶段结构的时滞捕食系统的正周期解

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摘要: 以捕食者有附加食物、食饵有阶段结构和避难所的时滞捕食系统为研究对象, 利用迭合度理论得到了该系统存在正周期解的充分条件, 并利用数值模拟验证了所得条件的正确性.

关键词: 阶段结构; 附加食物; 迭合度理论; 正周期解

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Periodic solution of delayed predator-prey model with stage-structured

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Abstract: A delayed predator-prey model with additional food for predator, stage-structured and refuge for prey is studied. We employ the coincidence degree theory to obtain the sufficient conditions of the positive periodic solution. In the end, the numerical simulation is given to show the validity of the obtained conditions.

Keywords: stage-structured; additional food; coincidence degree; positive periodic solution

0 引言

Lotka^[1] 和 Volterra^[2] 提出捕食者的数学模型后, 许多学者对其相关模型进行了改进, 并对相关定性问题进行了研究^[3-13]. 例如: 魏凤英等^[11] 研究了食饵具有阶段结构和避难机能的捕食者系统的分支及其稳定性问题; Srinivasu 等^[12] 研究了捕食者具有附加食饵的捕食者系统的相关性质, 并讨论了附加食饵的“数量”对系统的影响. 白玉珍等^[13] 研究了具有阶段结构和附加食饵与避难机能相结合的自治捕食系统的分支及其稳定性问题, 其研究的模型为:

$$\begin{cases} x_1' = ax_2(t) - bx_1(t) - \alpha x_1(t), \\ x_2' = \alpha x_1(t) - cx_2(t) - d_1 x_2^2(t) - \frac{k_1(1-m)e_1 x_2(t)y(t)}{a_1 + h_2 e_2 A' + h_1 e_1 x_2(t)}, \\ y' = \frac{k_2[(1-m)e_1 x_2(t-\tau) + e_2 A']}{a_1 + h_2 e_2 A' + h_1 e_1 x_2(t-\tau)} y(t-\tau) - ry(t). \end{cases} \quad (1)$$

其中: $x_1(t)$ 、 $x_2(t)$ 、 $y(t)$ 表示未成年食饵、成年食饵和捕食者在 t 时刻的种群密度; m 为避难参数, $m \in [0, 1]$; $k_1(1-m)$ 是捕食者的捕获率; h_2 表示捕食者处理单位数量的额外食物所需的时间; A' 表示额外食物量. 系统(1) 中的其他数学符号的生物意义见文献[13]. 为减少参数以便计算, 文献[13] 还将系统

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(1) 简化为了如下形式:

$$\begin{cases} u_1' = au_2(t) - bu_1(t) - \alpha u_1(t), \\ u_2' = \alpha u_1(t) - cu_2(t) - du_2^2(t) - \frac{(1-m)u_2(t)v(t)}{1+\beta\rho+u_2(t)}, \\ v' = \frac{k_2[(1-m)u_2(t-\tau)+\rho]}{1+\beta\rho+u_2(t-\tau)}v(t-\tau) - rv(t). \end{cases} \quad (2)$$

文献[13]考虑的是自治系统,且假设的系数均为正常数.而事实上,生态系统中很多因素是随时间不断变化的,如环境的周期性变化、繁殖的周期性变化等,因此研究非自治系统的周期解问题具有重要意义.本文考虑将系统(2)改进为非自治系统,并假设未成年食饵固有增长率 a 、未成年食饵死亡率 b 、成年食饵死亡率 c 、捕食者死亡率 r 、未成年食饵到成年食饵的成长率 α 和成年食饵内部竞争率 d 等参数为时间 t 的连续周期函数.由上述假设系统(2)可转化为如下非自治系统:

$$\begin{cases} u_1' = a(t)u_2(t) - b(t)u_1(t) - \alpha(t)u_1(t), \\ u_2' = \alpha(t)u_1(t) - c(t)u_2(t) - d(t)u_2^2(t) - \frac{(1-m)u_2(t)v(t)}{1+\beta\rho+u_2(t)}, \\ v' = \frac{k_2[(1-m)u_2(t-\tau)+\rho]}{1+\beta\rho+u_2(t-\tau)}v(t-\tau) - r(t)v(t). \end{cases} \quad (3)$$

以下本文将应用迭合度理论探讨系统(3)至少存在一个正周期解的充分条件.

1 主要结果及其证明

考虑到系统(3)的生态意义,本文只考虑初值 $u_1(0) > 0$ 、 $u_2(0) > 0$ 和 $v(0) > 0$ 的解.因为函数 $f(x) = \frac{Ax+B}{C+Dx}$ 的导数为 $f'(x) = \frac{AC-BD}{(C+Dx)^2}$,所以如下引理1显然成立.

引理 1 设函数 $f(x) = \frac{Ax+B}{C+Dx}$ (A, B, C, D 为正常数),当 $AC-BD > 0$ 时, $f(x)$ 在 $(0, +\infty)$ 上单调递增;当 $AC-BD < 0$ 时, $f(x)$ 在 $(0, +\infty)$ 上单调递减.

引理 2^[14] 设映射 L 为指标为零的Fredholm映射, N 在 $\bar{\Omega}$ 上是 L -紧的,并且设:

(a) 对任意的 $\lambda \in (0, 1)$,方程 $Lx = \lambda Nx$ 的解满足 $x \notin \partial\Omega \cap \text{Dom } L$;

(b) 对任意的 $x \in \partial\Omega \cap \text{Ker } L$, $QNx \neq 0$, $\deg\{JQN, \Omega \cap \text{Ker } L, 0\} \neq 0$,

则方程 $Lx = Nx$ 在 $\text{Dom } L \cap \bar{\Omega}$ 中至少存在一个解.

下文使用如下记号: $\bar{f} = \frac{1}{\omega} \int_0^\omega f(t)dt$, $f^l = \min_{t \in [0, \omega]} f(t)$, $f^u = \max_{t \in [0, \omega]} f(t)$, 其中 $f(t)$ 为连续的 ω -周期函数.

定理 1 系统(3)至少存在1个正周期解,若系统(3)满足以下3个条件:

(a) $(1-m)(1+\beta\rho) > \rho$;

(b) $\frac{k_2\rho}{1+\beta\rho} < r^l < r^u < k_2(1-m)$;

(c) $\frac{\alpha^l a^l}{b^u + \alpha^u} - c^u - d^u \frac{(1+\beta\rho)r^u - k_2\rho}{k_2(1-m) - r^u} > 0$.

证明 令 $\bar{u}_1(t) = \ln u_1(t)$, $\bar{u}_2(t) = \ln u_2(t)$, $\bar{v}_1(t) = \ln v_1(t)$, 则系统(3)可转化为

$$\begin{cases} \bar{u}_1' = \frac{a(t)\exp\{\bar{u}_2(t)\}}{\exp\{\bar{u}_1(t)\}} - b(t) - \alpha(t), \\ \bar{u}_2' = \frac{\alpha(t)\exp\{\bar{u}_1(t)\}}{\exp\{\bar{u}_2(t)\}} - c(t) - d(t)\exp\{\bar{u}_2(t)\} - \frac{(1-m)\exp\{\bar{v}(t)\}}{1+\beta\rho+\exp\{\bar{u}_2(t)\}}, \\ \bar{v}' = \frac{k_2[(1-m)\exp\{\bar{u}_2(t-\tau)\}+\rho]}{1+\beta\rho+\exp\{\bar{u}_2(t-\tau)\}} \frac{\exp\{\bar{v}(t-\tau)\}}{\exp\{\bar{v}(t)\}} - r(t). \end{cases} \quad (4)$$

如果系统(4)有 ω -周期解 $(\bar{u}_1^*(t), \bar{u}_2^*(t), \bar{v}^*(t))^T$, 则系统(3)有 ω -周期解 $(\exp \bar{u}_1^*(t), \exp \bar{u}_2^*(t), \exp \bar{v}^*(t))^T$, 因此只需证明系统(4)存在 ω -周期解即可. 取

$$X = Z = \{\tilde{w}(t) = (\tilde{u}_1(t), \tilde{u}_2(t), \tilde{v}(t)) \in C(R, R^3) \mid \tilde{w}(t + \omega) = \tilde{w}(t)\},$$

$$\|\tilde{w}\| = \max_{t \in [0, \omega]} |\tilde{u}_1(t)| + \max_{t \in [0, \omega]} |\tilde{u}_2(t)| + \max_{t \in [0, \omega]} |\tilde{v}(t)|.$$

显然, 在以上定义的范数下, X 和 Z 为 Banach 空间. 为了应用引理 2, 本文定义以下映射:

$$N\tilde{w} = A(t) = \begin{bmatrix} \frac{a(t)\exp\{\tilde{u}_2(t)\}}{\exp\{\tilde{u}_1(t)\}} - b(t) - \alpha(t) \\ \frac{\alpha(t)\exp\{\tilde{u}_1(t)\}}{\exp\{\tilde{u}_2(t)\}} - c(t) - d(t)\exp\{\tilde{u}_2(t)\} - \frac{(1-m)\exp\{\tilde{v}(t)\}}{1+\beta\rho + \exp\{\tilde{u}_2(t)\}} \\ \frac{k_2[(1-m)\exp\{\tilde{u}_2(t-\tau)\} + \rho]}{1+\beta\rho + \exp\{\tilde{u}_2(t-\tau)\}} \frac{\exp\{\tilde{v}(t-\tau)\}}{\exp\{\tilde{v}(t)\}} - r(t) \end{bmatrix},$$

$$L\tilde{w} = \tilde{w}'(t), P\tilde{w} = \frac{1}{\omega} \int_0^\omega \tilde{w}(t) dt, \tilde{w} \in X; Q\tilde{w} = \frac{1}{\omega} \int_0^\omega \tilde{w}(t) dt, \tilde{w} \in Z.$$

由空间 X, Z 和映射 L 的定义可知:

$$\text{Ker } L = \{\tilde{w}(t) \in X \mid \tilde{w}'(t) = 0\} = R^3,$$

$$\text{Im } L = \left\{ \tilde{w}(t) \in Z \mid \int_0^\omega \tilde{w}(t) dt = 0 \right\}, \dim \ker L = \text{codim Im } L = 3.$$

因为 $\text{Im } L$ 是 Z 中的闭子集, 所以 L 是指标为零的 Fredholm 映射. 根据映射 P 和 Q 的定义知, $P^2 = P$, $Q^2 = Q$, 所以 P 和 Q 为连续投影, 且有 $\text{Im } P = \text{Ker } L$, $\text{Im } L = \text{Ker } Q = \text{Im}(I - Q)$. 因此 $L|_{\text{Dom } L \cap \text{Ker } P} : (I - P)X \rightarrow \text{Im } L$ 可逆. 定义逆映射 $K_P : \text{Im } L \rightarrow \text{Dom } L \cap \text{Ker } P$. 若 $\tilde{w}(t) \in \text{Im } L$, 则 $\tilde{w}(t)$ 在 X 中的原像为 $\int_0^t \tilde{w}(s) ds$, 因此逆映射 K_P 可表示为 $(K_P \tilde{w})(t) = \int_0^t \tilde{w}(s) ds - \frac{1}{\omega} \int_0^\omega \int_0^t \tilde{w}(s) ds dt$. 于是有:

$$(QN)\tilde{w} = \frac{1}{\omega} \int_0^\omega A(t) dt,$$

$$(K_P(I - Q)N)\tilde{w} = \int_0^t A(s) ds - \frac{1}{\omega} \int_0^\omega \int_0^t A(s) ds dt + \left(\frac{1}{2} - \frac{t}{\omega} \right) \int_0^\omega A(s) ds.$$

因为映射 Q, N 和 K_P 为连续映射, 所以可知 QN 和 $K_P(I - Q)N$ 也是连续映射. 再由 Arzela-Ascoli 定理可知, 对任意 Ω 为 X 中的有界开集, $K_P(I - Q)N(\bar{\Omega})$ 是紧集, 且 $QN(\bar{\Omega})$ 有界, 所以 N 在 $\bar{\Omega}$ 是 L -紧的.

以下在定理 1 的条件下求解满足引理 2 的有界开集 Ω . 由方程 $Lx = \lambda Nx$, $\lambda \in (0, 1)$ 得

$$\begin{cases} \bar{u}_1' = \lambda \left[\frac{a(t)\exp\{\bar{u}_2(t)\}}{\exp\{\bar{u}_1(t)\}} - b(t) - \alpha(t) \right], \\ \bar{u}_2' = \lambda \left[\frac{\alpha(t)\exp\{\bar{u}_1(t)\}}{\exp\{\bar{u}_2(t)\}} - c(t) - d(t)\exp\{\bar{u}_2(t)\} - \frac{(1-m)\exp\{\bar{v}(t)\}}{1+\beta\rho + \exp\{\bar{u}_2(t)\}} \right], \\ \bar{v}' = \lambda \left[\frac{k_2[(1-m)\exp\{\bar{u}_2(t-\tau)\} + \rho]}{1+\beta\rho + \exp\{\bar{u}_2(t-\tau)\}} \frac{\exp\{\bar{v}(t-\tau)\}}{\exp\{\bar{v}(t)\}} - r(t) \right]. \end{cases} \quad (5)$$

设 $(\bar{u}_1(t), \bar{u}_2(t), \bar{v}(t))^T \in X$ 是系统(5)的解, 对系统(5)的两边同时在 $[0, \omega]$ 上积分得

$$\begin{cases} (\bar{b} + \bar{\alpha})\omega = \int_0^\omega \frac{a(t)\exp\{\bar{u}_2(t)\}}{\exp\{\bar{u}_1(t)\}} dt, \\ \bar{c}\omega = \int_0^\omega \left[\frac{\alpha(t)\exp\{\bar{u}_1(t)\}}{\exp\{\bar{u}_2(t)\}} - d(t)\exp\{\bar{u}_2(t)\} - \frac{(1-m)\exp\{\bar{v}(t)\}}{1+\beta\rho + \exp\{\bar{u}_2(t)\}} \right] dt, \\ \bar{r}\omega = \int_0^\omega \frac{k_2[(1-m)\exp\{\bar{u}_2(t-\tau)\} + \rho]}{1+\beta\rho + \exp\{\bar{u}_2(t-\tau)\}} \frac{\exp\{\bar{v}(t-\tau)\}}{\exp\{\bar{v}(t)\}} dt. \end{cases}$$

再对式(5)的两边取绝对值, 然后对其两边同时在 $[0, \omega]$ 上积分得

$$\left\{ \begin{aligned} \int_0^\omega |\bar{u}'_1| dt &\leq \int_0^\omega \left| \frac{a(t)\exp\{\bar{u}_2(t)\}}{\exp\{\bar{u}_1(t)\}} - b(t) - \alpha(t) \right| dt \leq 2(\bar{b} + \bar{\alpha})\omega, \\ \int_0^\omega |\bar{u}'_2| dt &\leq \int_0^\omega \left| \frac{\alpha(t)\exp\{\bar{u}_1(t)\}}{\exp\{\bar{u}_2(t)\}} - \frac{(1-m)\exp\{\bar{v}(t)\}}{1+\beta\rho+\exp\{\bar{u}_2(t)\}} \right| dt + \int_0^\omega d(t)\exp\{\bar{u}_2(t)\} dt + \\ &\quad \int_0^\omega c(t) dt \leq 2\bar{c}\omega, \\ \int_0^\omega |\bar{v}'| dt &\leq \int_0^\omega \left| \frac{k_2[(1-m)\exp\{\bar{u}_2(t-\tau)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(t-\tau)\}} \frac{\exp\{\bar{v}(t-\tau)\}}{\exp\{\bar{v}(t)\}} - r(t) \right| dt \leq 2\bar{r}\omega. \end{aligned} \right. \quad (6)$$

由于 $(\bar{u}_1(t), \bar{u}_2(t), \bar{v}(t))^T \in X$, 因此存在 $\xi_i, \eta_i \in [0, \omega]$, $i=1, 2, 3$, 且使:

$$\bar{u}_1(\xi_1) = \min_{t \in [0, \omega]} \bar{u}_1(t), \quad \bar{u}_2(\xi_2) = \min_{t \in [0, \omega]} \bar{u}_2(t), \quad \bar{v}(\xi_3) = \min_{t \in [0, \omega]} \bar{v}(t);$$

$$\bar{u}_1(\eta_1) = \max_{t \in [0, \omega]} \bar{u}_1(t), \quad \bar{u}_2(\eta_2) = \max_{t \in [0, \omega]} \bar{u}_2(t), \quad \bar{v}(\eta_3) = \max_{t \in [0, \omega]} \bar{v}(t).$$

首先, 根据定理 1 中的条件(a)、引理 1 以及系统(5) 中的第 3 式可得

$$\begin{aligned} r(\xi_3) &= \frac{k_2[(1-m)\exp\{\bar{u}_2(\xi_3-\tau)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(\xi_3-\tau)\}} \frac{\exp\{\bar{v}(\xi_3-\tau)\}}{\exp\{\bar{v}(\xi_3)\}} \geq \\ &\frac{k_2[(1-m)\exp\{\bar{u}_2(\xi_3-\tau)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(\xi_3-\tau)\}} \geq \frac{k_2[(1-m)\exp\{\bar{u}_2(\xi_2)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(\xi_2)\}}, \end{aligned}$$

于是有

$$r^u \geq \frac{k_2[(1-m)\exp\{\bar{u}_2(\xi_2)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(\xi_2)\}}. \quad (7)$$

再由式(7) 可得 $\exp\{\bar{u}_2(\xi_2)\} \leq \frac{(1+\beta\rho)r^u - k_2\rho}{k_2(1-m) - r^u} := B_1$. 由定理 1 中的条件(b) 有 $B_1 > 0$, 即 $\bar{u}_2(\xi_2) \leq$

$\ln B_1$, 进而由式(6) 可得

$$\bar{u}_2(t) \leq \bar{u}_2(\xi_2) + \int_0^\omega |\bar{u}'_2(t)| dt \leq \ln B_1 + 2\bar{c}\omega. \quad (8)$$

另外, 由定理 1 条件中的(a)、引理 1 以及系统(5) 中的第 3 式还可得

$$r(\eta_3) \leq \frac{k_2[(1-m)\exp\{\bar{u}_2(\eta_3-\tau)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(\eta_3-\tau)\}} \leq \frac{k_2[(1-m)\exp\{\bar{u}_2(\eta_2)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(\eta_2)\}},$$

于是有

$$r^l \leq \frac{k_2[(1-m)\exp\{\bar{u}_2(\eta_2)\} + \rho]}{1+\beta\rho+\exp\{\bar{u}_2(\eta_2)\}}. \quad (9)$$

再由式(9) 可得 $\exp\{\bar{u}_2(\eta_2)\} \geq \frac{(1+\beta\rho)r^l - k_2\rho}{k_2(1-m) - r^l} := B_2$. 由定理 1 中的条件(b) 有 $B_2 > 0$, 即 $\bar{u}_2(\eta_2) \geq$

$\ln B_2$, 进而由式(6) 可得

$$\bar{u}_2(t) \geq \bar{u}_2(\eta_2) - \int_0^\omega |\bar{u}'_2(t)| dt \geq \ln B_2 - 2\bar{c}\omega. \quad (10)$$

其次, 由系统(5) 中的第 1 式可得 $\frac{a(\xi_1)\exp\{\bar{u}_2(\xi_1)\}}{\exp\{\bar{u}_1(\xi_1)\}} \geq b^l + \alpha^l$, 因此

$$\exp\{\bar{u}_1(\xi_1)\} \leq \frac{a(\xi_1)\exp\{\bar{u}_2(\xi_1)\}}{b^l + \alpha^l} \leq \frac{a^u \exp\{\bar{u}_2(\eta_2)\}}{b^l + \alpha^l} \leq \frac{a^u \exp\{\ln B_1 + 2\bar{c}\omega\}}{b^l + \alpha^l} = \frac{a^u B_1 e^{2\bar{c}\omega}}{b^l + \alpha^l} := A_1.$$

于是有 $\bar{u}_1(\xi_1) \leq \ln A_1$. 再由式(6) 可得

$$\bar{u}_1(t) \leq \bar{u}_1(\xi_1) + \int_0^\omega |\bar{u}'_1(t)| dt \leq \ln A_1 + 2(\bar{b} + \bar{\alpha})\omega. \quad (11)$$

另外, 由系统(5) 中的第 1 式还可得 $\frac{a(\eta_1)\exp\{\bar{u}_2(\eta_1)\}}{\exp\{\bar{u}_1(\eta_1)\}} \leq b^u + \alpha^u$, 因此

$$\exp\{\bar{u}_1(\eta_1)\} \geq \frac{a(\eta_1)\exp\{\bar{u}_2(\eta_1)\}}{b'' + \alpha''} \geq \frac{a^l \exp\{\bar{u}_2(\eta_1)\}}{b'' + \alpha''} \geq \frac{a^l \exp\{\ln B_2 - 2\bar{c}\omega\}}{b'' + \alpha''} = \frac{a^l B_2 e^{-2\bar{c}\omega}}{b'' + \alpha''} := A_2.$$

于是有 $\bar{u}_1(\eta_1) \geq \ln A_2$. 再由式(6) 可得

$$\bar{u}_1(t) \geq \bar{u}_1(\eta_1) - \int_0^\omega |\bar{u}'_1(t)| dt \geq \ln A_2 - 2(\bar{b} + \bar{\alpha})\omega. \quad (12)$$

最后,由系统(5) 中的第2 式可得 $\frac{(1-m)\exp\{\bar{v}(\xi_2)\}}{1+\beta\rho+\exp\{\bar{u}'_2(\xi_2)\}} \leq \frac{\alpha(t)\exp\{\bar{u}_1(\xi_2)\}}{\exp\{\bar{u}_2(\xi_2)\}}$, 因此

$$\begin{aligned} \exp\{\bar{v}(\xi_3)\} &\leq \exp\{\bar{v}(\xi_2)\} \leq \frac{\alpha(t)\exp\{\bar{u}_1(\xi_2)\} (1+\beta\rho+\exp\{\bar{u}_2(\xi_2)\})}{(1-m)\exp\{\bar{u}_2(\xi_2)\}} \leq \\ &\frac{\alpha'' \exp\{\ln A_1 + 2(\bar{b} + \bar{\alpha})\omega\} (1+\beta\rho+\exp\{\ln B_2 + 2\bar{c}\omega\})}{(1-m)\exp\{\ln B_2 - 2\bar{c}\omega\}} \leq \\ &\frac{\alpha'' A_1 e^{2(\bar{b} + \bar{\alpha})\omega} (1+\beta\rho+B_2 e^{2\bar{c}\omega})}{(1-m)B_2 e^{-2\bar{c}\omega}} := C_1, \end{aligned}$$

于是有 $\bar{v}(\xi_3) \leq \ln C_1$. 再由式(6) 可得

$$\bar{v}(t) \leq \bar{v}(\xi_3) + \int_0^\omega |\bar{v}(t)| dt \leq \ln C_1 + 2\bar{r}\omega. \quad (13)$$

另外,由系统(5) 中的第1 式和第2 式分别可得:

$$\frac{\exp\{\bar{u}_2(\xi_2)\}}{\exp\{\bar{u}_1(\xi_2)\}} \leq \frac{\exp\{\bar{u}_2(\xi_1)\}}{\exp\{\bar{u}_1(\xi_1)\}} \leq \frac{b'' + \alpha''}{a^l}, \quad (14)$$

$$\frac{\alpha(t)\exp\{\bar{u}_1(\xi_2)\}}{\exp\{\bar{u}_2(\xi_2)\}} - c(\xi_2) - d(\xi_2)\exp\{\bar{u}_2(\xi_2)\} - \exp\{\bar{v}(\eta_3)\} \leq 0. \quad (15)$$

由式(14) 和式(15) 可得

$$\exp\{\bar{v}(\eta_3)\} \geq \frac{\alpha(t)\exp\{\bar{u}_1(\xi_2)\}}{\exp\{\bar{u}_2(\xi_2)\}} - c(\xi_2) - d(\xi_2)\exp\{\bar{u}_2(\xi_2)\} \geq \frac{a^l a^l}{b'' + \alpha''} - c'' - d'' B_1 := C_2.$$

由定理1 中的条件(3) 知 $C_2 > 0$, 即 $\bar{v}(\eta_3) \geq \ln C_2$. 再由式(6) 可得

$$\bar{v}(t) \geq \bar{v}(\eta_3) - \int_0^\omega |\bar{v}(t)| dt \geq \ln C_2 - 2\bar{r}\omega. \quad (16)$$

综合式(8)、(10)、(11)、(12)、(13) 和式(16) 可得:

$$\begin{aligned} \max_{t \in [0, \omega]} |\bar{u}_2(t)| &\leq \max\{|\ln B_1 + 2\bar{c}\omega|, |\ln B_2 - 2\bar{c}\omega|\} := H_1, \\ \max_{t \in [0, \omega]} |\bar{u}_1(t)| &\leq \max\{|\ln A_1 + 2(\bar{b} + \bar{\alpha})\omega|, |\ln A_2 - 2(\bar{b} + \bar{\alpha})\omega|\} := H_2, \\ \max_{t \in [0, \omega]} |\bar{v}(t)| &\leq \max\{|\ln C_2 - 2\bar{r}\omega|, |\ln C_2 - 2\bar{r}\omega|\} := H_3. \end{aligned}$$

由定理1 中的条件知代数方程组

$$\begin{cases} \frac{\bar{a}u_2}{u_1} - \bar{b} - \bar{\alpha} = 0, \\ \frac{\bar{a}u_1}{u_2} - \bar{c} - \bar{d}u_2 - \frac{(1-m)v}{1+\beta\rho+u_2} = 0, \\ \frac{k_2[(1-m)u_2+\rho]}{1+\beta\rho+u_2} - \bar{r} = 0 \end{cases} \quad (17)$$

有唯一正解: $u_2^* = \frac{(1+\beta\rho)\bar{r} - k_2\rho}{k_2(1-m) - \bar{r}}$, $u_1^* = \frac{\bar{a}u_2^*}{\bar{b} + \bar{\alpha}}$, $v^* = \frac{(1+\beta\rho)u_2^*}{(1-m)} \left(\frac{\bar{a}u_1^*}{u_2^*} - \bar{c} - \bar{d}u_2^* \right)$. 令 $H = H_1 + H_2 + H_3 + H_4$, 其中 H_4 充分大且 $|\ln u_1^*| + |\ln u_2^*| + |\ln v^*| \leq H_4$. 再令 $\Omega = \{\tilde{\mathbf{w}} = (u_1(t), u_2(t), v(t))^T \in X \mid \|\tilde{\mathbf{w}}\| < H\}$, 则可知 Ω 满足引理2 中的条件(a).

下面证明 Ω 满足引理2 中的条件(b). 当 $(u_1(t), u_2(t), v(t)) \in \partial\Omega \cap \text{Ker } L$ 时, $(u_1(t), u_2(t), v(t))$ 是常向量, 且 $\|(u_1(t), u_2(t), v(t))\| = H$, 所以 $QNx \neq 0$. 因为 $\text{Im } Q = \text{Ker } L = \mathbb{R}^3$, 所以同构映射 J 可

取恒等映射. 由于式(17) 存在唯一正解, 因此直接计算即可得

$$\deg\{JQN, \Omega \cap \text{Ker } L, 0\} = \text{sgn}\left(-\frac{\bar{a}u_2^*(1-m)}{(1+\beta\rho+u_2^*)(u_1^*)^2}\right) = -1 \neq 0.$$

综上, $Lx = Nx$ 在 $\text{Dom } L \cap \bar{\Omega}$ 中至少存在一个周期解, 即系统(4) 有正周期解 $(\bar{u}_1^*(t), \bar{u}_2^*(t), \bar{v}^*(t))^T$, 所以系统(3) 有相应的正周期解 $(\exp\{\bar{u}_1^*(t)\}, \exp\{\bar{u}_2^*(t)\}, \exp\{\bar{v}^*(t)\})^T$.

定理 2 系统(3) 至少存在一个正周期解, 若系统(3) 满足以下 3 个条件:

(a) $(1-m)(1+\beta\rho) < \rho$;

(b) $k_2(1-m) < r^l < r^u < \frac{k_2\rho}{1+\beta\rho}$;

(c) $\frac{\alpha^l a^l}{b^u + \alpha^u} - c^u - d^u \frac{(1+\beta\rho)r^l - k_2\rho}{k_2(1-m) - r^l} > 0$.

证明 因证明方法与定理 1 相似, 且只需注意函数的单调性即可, 因此本文在此省略.

2 数值模拟

从定理 1 可看出, 系统(3) 存在正周期解的充分条件与时滞变量 τ 无关, 因此本文对 $\tau=0$ 时的系统进行数值模拟. 令系统(3) 取如下参数: $\rho=2, \beta=3, k_2=0.5, m=\frac{1}{3}, a(t)=2+0.2\sin 2t, b(t)=0.5+0.05\sin 2t, \alpha(t)=2+\frac{\cos 2t}{10}, c(t)=0.09+0.01\cos 2t, d(t)=0.15+0.03\sin 2t, r(t)=0.22+0.01\sin 2t$. 将上述参数代入定理 1 的 3 个条件中得:

(a) $(1-m)(1+\beta\rho) = \frac{14}{3} > \rho=2$;

(b) $\frac{k_2\rho}{1+\beta\rho}(0.1429) < r^l(0.21) < r^u(0.23) < k_2(1-m)=0.3333$;

(c) $\frac{\alpha^l a^l}{b^u + \alpha^u} - c^u - d^u \frac{(1+\beta\rho)r^u - k_2\rho}{k_2 - r^u} = 0.1273 > 0$.

由(a)、(b)、(c)可知, 所取的参数满足定理 1 的 3 个条件. 在上述参数下系统(3)的解如图 1 所示. 由图 1 可知, 系统(3)的解是周期解.

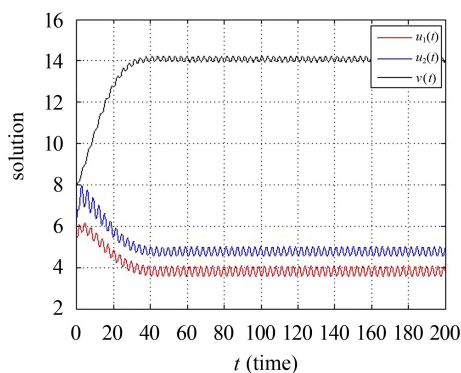


图 1 系统(3)的正周期解

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