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# 条件最优的两步迭代法及 Jarratt 变形方法

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**摘要:** 以  $y_n = x_n - \theta \frac{f(x_n)}{f'(x_n)}$  ( $0 < \theta \leq 1$ ) 为基础, 构造了一类新的带有参数的条件最优的两步迭代方法, 其收敛阶数可达到四阶, 且符合 Kung-Traub 猜想 ( $n=3$  情形). 另外, 该方法包含了一些已有的迭代法, 尤其包含了 Jarratt 方法. 数值验证表明, 本文方法优于牛顿迭代法及一些已有的方法, 具有较好的有效性和可行性.

**关键词:** 牛顿迭代法; 非线性方程; 迭代方法; 条件最优

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## A conditional optimal two-step iterative method and a variant of Jarratt method

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**Abstract:** Based on  $y_n = x_n - \theta \frac{f(x_n)}{f'(x_n)}$  ( $0 < \theta \leq 1$ ), a new conditional optimal two-step iterative method with one-parameter for solving nonlinear equations is presented. The order of convergence arrives to at least four and agrees with the conjecture of Kung-Traub for the case  $n=3$ . Moreover, this method contains some existing methods especially contains the Jarratt method. Several numerical results are shown that this method is superior to Newton's method and other existing methods. Therefore our method is more efficient and performs better than other methods.

**Keywords:** Newton's method; nonlinear equation; iterative method; conditional optimal

## 0 引言

非线性方程在数值分析中具有广泛的应用性, 但一般难以直接求解, 通常只能采用数值方法求其近似解; 为此, 本文考虑利用迭代法解非线性方程  $f(x)=0$  的单根问题. 众所周知, 经典的牛顿迭代法<sup>[1]</sup> 是基本的二阶迭代方法, 其公式为

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}. \quad (1)$$

Jarratt 方法<sup>[2]</sup> 作为四阶收敛方法, 被广泛应用于求解非线性方程及方程组的单根问题, 其计算公式为

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{3f'(y_n) + f'(x_n)}{6f'(y_n) - 2f'(x_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases}$$

Kung-Traub 猜想<sup>[3]</sup>指出,一个含有  $n$  个函数值的不带记忆的多步迭代法收敛阶最多可达到  $2^{n-1}$ . 由于牛顿迭代法(1)符合 Kung-Traub 猜想( $n=2$  情形),因此一些研究者在牛顿迭代法的基础上构造出了许多新的迭代方法<sup>[4-13]</sup>;但是,在计算过程中,这些方法中大多仍然需要计算函数的二阶导数. 为了避免计算函数的二阶导数,Darvishi 等<sup>[5]</sup>建立了一类三阶牛顿型迭代法;Kou 等<sup>[6]</sup>构造了一类复合四阶迭代法;Chun<sup>[7]</sup>构造了一类含有参数的三阶迭代法,并且该方法只需 3 个函数计算量;Liu X L 等<sup>[8]</sup>在牛顿迭代法的基础上建立了如下一类一般的高阶迭代方法:

$$\begin{cases} z_n = \phi_p(x_n), \\ x_{n+1} = z_n - H(x_n, y_n) \frac{f(z_n)}{f'(x_n)}, \end{cases}$$

其中  $y_n = x_n - \frac{f(x_n)}{f'(x_n)}$ ,  $\phi_p(\cdot)$  为任意  $p$  阶迭代法,  $H(x_n, y_n)$  为一待定的二元函数;此外, Liu X L 等<sup>[9]</sup>还构造了一类不含导数的一般迭代方法:

$$\begin{cases} z_n = \phi_p(x_n), \\ x_{n+1} = z_n - h(\mu_n) \frac{f(z_n)}{f'(x_n)}, \end{cases}$$

其中  $\mu_n = \frac{f(y_n)}{f(x_n)}$ ,  $y_n = x_n - \frac{f(x_n)}{f'(x_n)}$ ,  $\phi_p(\cdot)$  为任意  $p$  阶迭代法,  $h(\cdot)$  为一充分光滑函数且不含函数  $f$  的任意阶导数. 在上述研究的基础上,本文构造了一类条件最优的两步迭代方法,该方法包含一待定二元函数  $H(x, y)$  且以  $y_n = x_n - \theta \frac{f(x_n)}{f'(x_n)}$  ( $0 < \theta \leq 1$ ) 作为第一步迭代,其收敛阶数至少可达到四阶,符合 Kung-Traub 猜想( $n=3$  情形).

1 收敛性分析

考虑如下迭代形式:

$$\begin{cases} y_n = x_n - \theta \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - H(x_n, y_n) \frac{f(x_n)}{f'(x_n)}. \end{cases} \tag{2}$$

其中  $\theta$  为参数,满足  $0 < \theta \leq 1$ ,  $H(x_n, y_n)$  为一待定二元函数. 令  $\alpha$  为非线性方程  $f(x)=0$  的单根,  $c_k = f^{(k)}(\alpha)/(k!f'(\alpha))$ ,  $k=2,3,\cdots$ . 假设  $H(x, y)$  充分光滑,其在  $(\alpha, \alpha)$  处的函数值及其各阶导数值简记为  $H, H_x, H_y, H_{xx}, \cdots$ . 下面定理中所用到的条件如下:

- (C1)  $H=1$ ;
- (C2)  $H_x + H_y(1-\theta) = c_2$ ;
- (C3)  $\theta H_y c_2 + \frac{1}{2}H_{xx} + H_{xy}(1-\theta) + \frac{1}{2}H_{yy}(1-\theta)^2 = 2c_3 - c_2^2$ .

**定理 1** 设  $f: D \subset \mathbf{R} \rightarrow \mathbf{R}$  为开区间  $D$  上的充分光滑函数,  $\alpha \in D$  为函数  $f(x)$  的简单零点且  $x_0$  充分接近于  $\alpha$ , 则下列结论成立:

- 1) 若  $H(x, y)$  满足条件(C1)–(C2), 则迭代法(2)至少为三阶收敛;
- 2) 若  $H(x, y)$  满足条件(C1)–(C3), 则迭代法(2)至少为四阶收敛.

**证明** 令  $e_n = x_n - \alpha$ , 将函数  $f(x)$  在  $\alpha$  处泰勒展开, 可得

$$\begin{aligned} f(x_n) &= f'(\alpha)[e_n + c_2 e_n^2 + c_3 e_n^3 + c_4 e_n^4 + c_5 e_n^5 + c_6 e_n^6 + O(e_n^7)], \\ f'(x_n) &= f'(\alpha)[1 + 2c_2 e_n + 3c_3 e_n^2 + 4c_4 e_n^3 + 5c_5 e_n^4 + 6c_6 e_n^5 + O(e_n^6)], \end{aligned}$$

因此有:

$$\begin{aligned} \frac{f(x_n)}{f'(x_n)} &= e_n - c_2 e_n^2 + 2(c_2^2 - c_3) e_n^3 + (7c_2 c_3 - 4c_2^3 - 3c_4) e_n^4 + (10c_2 c_4 - 20c_2^2 c_3 - 4c_5 + 6c_3^2 + \\ &\quad 8c_2^4) e_n^5 + (13c_2 c_5 - 5c_6 - 28c_2^2 c_4 + 17c_3 c_4 + 52c_2^3 c_3 - 33c_2 c_3^2 + 16c_2^5) e_n^6 + O(e_n^7), \\ y_n - \alpha &= (1 - \theta) e_n + \theta c_2 e_n^2 + \theta(2c_3 - 2c_2^2) e_n^3 + \theta(4c_2^3 + 3c_4 - 7c_2 c_3) e_n^4 + \theta(20c_2^2 c_3 + 4c_5 - 10c_2 c_4 - \\ &\quad 6c_3^2 - 8c_2^4) e_n^5 + \theta(5c_6 - 13c_2 c_5 + 28c_2^2 c_4 - 17c_3 c_4 - 52c_2^3 c_3 + 33c_2 c_3^2 - 16c_2^5) e_n^6 + O(e_n^7). \end{aligned}$$

利用泰勒定理,将函数  $H(x_n, y_n)$  在点  $(\alpha, \alpha)$  处展开,可得

$$\begin{aligned} H(x_n, y_n) &= H + [H_x + H_y(1 - \theta)] e_n + [\theta H_y c_2 + \frac{1}{2} H_{xx} + H_{xy}(1 - \theta) + \frac{1}{2} H_{yy}(1 - \theta)^2] e_n^2 + \\ &\quad [\theta H_y(2c_3 - 2c_2^2) + H_{xy} \theta c_2 + H_{yy}(1 - \theta) \theta c_2 + \frac{1}{6} H_{xxx} + \frac{1}{2} H_{xxy}(1 - \theta) + \frac{1}{2} H_{xyy}(1 - \theta)^2 + \\ &\quad \frac{1}{6} H_{yyy}(1 - \theta)^3] e_n^3 + [\theta H_y(4c_2^3 + 3c_4 - 7c_2 c_3) + H_{xy} \theta(2c_3 - 2c_2^2) + \frac{1}{2} H_{yy} \theta^2 c_2^2 + \\ &\quad H_{yy}(1 - \theta) \theta(2c_3 - 2c_2^2) + \frac{1}{2} H_{xxy} \theta c_2 + H_{xyy}(1 - \theta) \theta c_2 + \frac{1}{3} H_{yyy} \theta(1 - \theta)^2 c_2 + \frac{1}{24} H_{xxxx} + \\ &\quad \frac{1}{6} H_{xxxy}(1 - \theta) + \frac{1}{4} H_{xxyy}(1 - \theta)^2 + \frac{1}{6} H_{xyyy}(1 - \theta)^3 + \frac{1}{24} H_{yyyy}(1 - \theta)^4] e_n^4 + O(e_n^5). \end{aligned}$$

进一步,可知

$$\begin{aligned} H(x_n, y_n) \frac{f(x_n)}{f'(x_n)} &= H e_n + [H_x + H_y(1 - \theta) - H c_2] e_n^2 + [H(2c_2^2 - 2c_3) - c_2 H_x - \\ &\quad c_2 H_y(1 - \theta) + \theta H_y c_2 + \frac{1}{2} H_{xx} + H_{xy}(1 - \theta) + \frac{1}{2} H_{yy}(1 - \theta)^2] e_n^3 + [H(7c_2 c_3 - 3c_4 - 4c_2^3) + \\ &\quad (2c_2^2 - 2c_3)(H_x + H_y(1 - \theta)) - H_y \theta c_2^2 - \frac{1}{2} c_2 H_{xx} - c_2 H_{xy}(1 - \theta) - \frac{1}{2} c_2 H_{yy}(1 - \theta)^2 + \\ &\quad \theta H_y(2c_3 - 2c_2^2) + H_{xy} \theta c_2 + H_{yy}(1 - \theta) \theta c_2 + \frac{1}{6} H_{xxx} + \frac{1}{2} H_{xxy}(1 - \theta) + \\ &\quad \frac{1}{2} H_{xyy}(1 - \theta)^2 + \frac{1}{6} H_{yyy}(1 - \theta)^3] e_n^4 + O(e_n^5). \end{aligned}$$

若条件(C1)–(C2) 成立,则

$$e_{n+1} = \{2c_3 - c_2^2 - [\theta H_y c_2 + \frac{1}{2} H_{xx} + H_{xy}(1 - \theta) + \frac{1}{2} H_{yy}(1 - \theta)^2]\} e_n^3 + O(e_n^4).$$

若条件(C1)–(C3) 成立,则

$$\begin{aligned} e_{n+1} &= \{(c_2^3 + 3c_4 - 3c_2 c_3) - [\theta H_y(2c_3 - 2c_2^2) + H_{xy} \theta c_2 + H_{yy}(1 - \theta) \theta c_2 + \frac{1}{6} H_{xxx} + \\ &\quad \frac{1}{2} H_{xxy}(1 - \theta) + \frac{1}{2} H_{xyy}(1 - \theta)^2 + \frac{1}{6} H_{yyy}(1 - \theta)^3]\} e_n^4 + O(e_n^5). \end{aligned}$$

证毕.

这里,如果条件(C1)–(C2) 成立,令  $H(x, y) = \frac{\xi f'(x) + \beta f'(y)}{\gamma f'(x) + \eta f'(y)}$ , 其中  $\xi, \eta, \gamma, \beta$  为未知常数,则

$$\begin{cases} \frac{\xi + \beta}{\gamma + \eta} = 1, \\ \frac{\xi \eta - \beta \gamma}{(\gamma + \eta)^2} \cdot 2\theta = 1 \end{cases} \quad \text{成立. 当 } \theta = 1, H(x, y) = \frac{2f'(x)}{f'(x) + f'(y)} \text{ 时,方法(2) 可写为}$$
$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{2f'(x_n)}{f'(x_n) + f'(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases}$$

此方法即为文献[10]中所给出的三阶迭代法. 当  $\theta = \frac{1}{2}$ ,  $H(x, y) = \frac{3f'(x) - f'(y)}{f'(x) + f'(y)}$  时, 方法(2) 为一新的三阶迭代法:

$$\begin{cases} y_n = x_n - \frac{1}{2} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{3f'(x_n) - f'(y_n)}{f'(x_n) + f'(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \quad (3)$$

当  $\theta = \frac{1}{3}$ ,  $H(x, y) = \frac{f'(x) + f'(y)}{-2f'(x) + 4f'(y)}$  时, 方法(2) 为一新的三阶迭代法:

$$\begin{cases} y_n = x_n - \frac{1}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{f'(x_n) + f'(y_n)}{-2f'(x_n) + 4f'(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \quad (4)$$

当  $\theta = \frac{2}{3}$ ,  $H(x, y) = \frac{5f'(x) - f'(y)}{2f'(x) + 2f'(y)}$  时, 方法(2) 为一新的三阶迭代法:

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{5f'(x_n) - f'(y_n)}{2f'(x_n) + 2f'(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \quad (5)$$

如果条件(C1)–(C2) 成立, 令  $H(x, y) = \frac{\xi f'^2(x) + \eta f'(x)f'(y) + \gamma f'^2(y)}{\xi_1 f'^2(x) + \eta_1 f'(x)f'(y) + \gamma_1 f'^2(y)}$ , 其中  $\xi, \eta, \gamma, \xi_1, \eta_1, \gamma_1$  为任意常数, 则下列条件成立:

$$\begin{cases} \frac{\xi + \eta + \gamma}{\xi_1 + \eta_1 + \gamma_1} = 1, \\ \frac{2\xi + \eta}{\xi_1 + \eta_1 + \gamma_1} + (1 - \theta) \frac{\eta + 2\gamma}{\xi_1 + \eta_1 + \gamma_1} - \frac{(\xi + \eta + \gamma)(2\xi_1 + \eta_1)}{(\xi_1 + \eta_1 + \gamma_1)^2} + (1 - \theta) \frac{(\xi + \eta + \gamma)(2\gamma_1 + \eta_1)}{(\xi_1 + \eta_1 + \gamma_1)^2} = \frac{1}{2}. \end{cases}$$

当  $\theta = 1$ ,  $H(x, y) = \frac{f'^2(x) + f'^2(y)}{f'(x)f'(y) + f'^2(y)}$  时, 方法(2) 为一新的三阶迭代法:

$$\begin{cases} y_n = x_n - \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{f'^2(x_n) + f'^2(y_n)}{f'(x_n)f'(y_n) + f'^2(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases}$$

当  $\theta = \frac{1}{2}$ ,  $H(x, y) = \frac{5f'^2(x) + f'^2(y)}{f'^2(x) + 2f'(x)f'(y) + 2f'^2(y)}$  时, 方法(2) 为一新的三阶迭代法:

$$\begin{cases} y_n = x_n - \frac{1}{2} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{5f'^2(x_n) + f'^2(y_n)}{f'^2(x_n) + 2f'(x_n)f'(y_n) + 2f'^2(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \quad (6)$$

当  $\theta = \frac{1}{3}$ ,  $H(x, y) = \frac{4f'^2(x) + f'(x)f'(y) + f'^2(y)}{2f'^2(x) + 2f'(x)f'(y) + 2f'^2(y)}$  时, 方法(2) 为一新的三阶迭代法:

$$\begin{cases} y_n = x_n - \frac{1}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{4f'^2(x_n) + f'(x_n)f'(y_n) + f'^2(y_n)}{2f'^2(x_n) + 2f'(x_n)f'(y_n) + 2f'^2(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \quad (7)$$

当  $\theta = \frac{2}{3}$ ,  $H(x, y) = \frac{4f'^2(x) + 3f'(x)f'(y) + f'^2(y)}{f'^2(x) + 7f'^2(y)}$  时, 方法(2) 为一新的三阶迭代方法:

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{4f'^2(x_n) + 3f'(x_n)f'(y_n) + f'^2(y_n)}{f'^2(x_n) + 7f'^2(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \tag{8}$$

若条件(C1)–(C3) 成立, 令  $H(x, y) = \frac{\xi f'(x) + \beta f'(y)}{\gamma f'(x) + \eta f'(y)}$ , 其中  $\xi, \eta, \gamma, \beta$  为未知常数, 则下列条件成立:

$$\begin{cases} \frac{\xi + \beta}{\gamma + \eta} = 1, \\ \frac{\xi \eta - \beta \gamma}{(\gamma + \eta)^2} \cdot 2\theta = 1, \\ \frac{\xi \eta - \beta \gamma}{(\gamma + \eta)^2} \cdot \theta(2 - \theta) = \frac{2}{3}, \\ \frac{\xi \eta - \beta \gamma}{(\gamma + \eta)^2} (4 - 6\theta) - \frac{\xi \eta - \beta \gamma}{(\gamma + \eta)^3} [4\gamma + 8\eta(1 - \theta) - 4\eta(1 - \theta)^2] = -1. \end{cases}$$

由此可得  $\theta = \frac{2}{3}$ , 进一步可知  $\xi, \eta, \gamma$  以及  $\beta$  满足条件  $\begin{cases} \eta = -3\gamma, \\ \xi + \beta = -2\gamma. \end{cases}$  当  $H(x, y) = \frac{3f'(y) + f'(x)}{6f'(y) - 2f'(x)}$  时,

方法(2) 为四阶迭代方法:

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{3f'(y_n) + f'(x_n)}{6f'(y_n) - 2f'(x_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \tag{9}$$

此方法即为著名的 Jarratt 方法. 当  $H(x, y) = \frac{f'(x) + f'(y)}{3f'(y) - f'(x)}$  时, 方法(2) 为四阶迭代方法:

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{f'(x_n) + f'(y_n)}{3f'(y_n) - f'(x_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases} \tag{10}$$

当  $H(x, y) = \frac{5f'(x) - 3f'(y)}{3f'(y) - f'(x)}$  时, 方法(2) 为四阶迭代方法:

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{5f'(x_n) - 3f'(y_n)}{3f'(y_n) - f'(x_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases}$$

当  $H(x, y) = \frac{3f'(x) - 5f'(y)}{f'(x) - 3f'(y)}$  时, 迭代方法(2) 为四阶迭代方法:

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{3f'(x_n) - 5f'(y_n)}{f'(x_n) - 3f'(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases}$$

当  $H(x, y) = \frac{2f'(x) - 4f'(y)}{f'(x) - 3f'(y)}$  时, 迭代方法(2) 为四阶迭代方法:

$$\begin{cases} y_n = x_n - \frac{2}{3} \frac{f(x_n)}{f'(x_n)}, \\ x_{n+1} = x_n - \frac{2f'(x_n) - 5f'(y_n)}{f'(x_n) - 3f'(y_n)} \frac{f(x_n)}{f'(x_n)}. \end{cases}$$

从上述分析过程可知, 只要条件给定,  $\theta$  和  $H(x, y)$  选定, 那么迭代方法(2) 就能确定, 并且所得到的方法收敛阶数最高可达到四阶. 另外, 每次迭代过程中只需 3 次函数计算量, 因此符合 Kung-Traub 猜

想( $n=3$  情形).

2 实验验证

利用迭代方法(2) 求解非线性方程,并通过数值结果验证本文方法的有效性. 在求解过程中,所有计算均是通过 Microsoft visual studio 2008 得到,迭代终止条件为  $|x_{n+1} - x_n| < \sqrt{\epsilon}$  和  $|f(x_{n+1})| < \sqrt{\epsilon}$ , 其中  $\epsilon = 10^{-28}$ . 实验函数如下:

$$f_1(x) = x^3 + 4x^2 - 10, \alpha = 1.365\ 230\ 013\ 414\ 10;$$
$$f_2(x) = x^2 + e^x - 3x + 2, \alpha = 0.257\ 530\ 285\ 439\ 86;$$
$$f_3(x) = \cos x - x, \alpha = 0.739\ 085\ 133\ 215\ 16;$$
$$f_4(x) = (x - 1)^3 - 1, \alpha = 2;$$
$$f_5(x) = x^3 - 10, \alpha = 2.154\ 434\ 690\ 031\ 88;$$
$$f_6(x) = e^{x^2+7x-30} - 1, \alpha = 3.$$

数值结果如表 1 和表 2 所示,其中表 1 为四阶迭代法(10)(H1)、三阶迭代法(3)(H2)、三阶迭代法(6)(H3)和 Jarratt 方法(9)(JM)与牛顿迭代法(NM)比较所得到的数值,表 2 为三阶迭代法(4)(H4)、(7)(H5)、(5)(H6)和(8)(H7)与牛顿迭代法比较所得的数值. 在表 1 和表 2 中,  $x_0$  表示迭代初值,  $N$  表示迭代次数, NOFE 表示每次迭代总的函数计算次数.

表 1 迭代法 H1、H2、H3 和 Jarratt 方法与牛顿迭代法的数值结果

$f(x)$	$x_0$	N					NOFE					Root
		NM	JM	H1	H2	H3	NM	JM	H1	H2	H3	
$f_1(x)$	-0.5	112	14	8	11	52	224	42	24	33	156	$\alpha = 1.365\ 230\ 013\ 414\ 10$
	1	5	3	4	4	22	10	9	12	12	66	
	1.5	4	2	4	3	22	8	6	12	9	66	
	2	5	3	4	4	23	10	9	12	12	69	
$f_2(x)$	-0.3	54	51	9	25	39	108	153	27	75	117	$\alpha = 0.257\ 530\ 285\ 439\ 86$
	2	5	3	5	3	23	10	9	15	9	69	
	3	6	3	5	4	23	12	9	15	12	69	
	0.5	4	2	3	3	22	8	6	9	9	66	
	1	4	2	4	3	22	8	6	12	9	66	
	0.25	3	2	3	2	20	6	6	9	6	60	
	0.1	4	2	3	2	22	8	6	9	6	66	
$f_3(x)$	1	4	2	4	3	21	8	6	12	9	63	$\alpha = 0.739\ 085\ 133\ 215\ 16$
	0.5	4	2	4	3	21	8	6	12	9	63	
	1.7	4	3	4	3	22	8	9	12	9	66	
$f_4(x)$	-0.3	5	3	4	4	22	10	9	12	12	66	$\alpha = 2$
	3.5	7	4	5	5	23	14	13	15	15	69	
	2.5	6	3	5	4	21	12	9	15	12	63	
	1.5	7	4	4	22	22	14	12	12	66	66	
$f_5(x)$	1.7	6	3	5	4	22	12	9	15	12	66	$\alpha = 2.154\ 434\ 690\ 031\ 88$
	1.5	6	4	6	5	22	12	12	18	15	66	
	2	4	3	5	4	22	8	9	15	12	66	
	2.5	5	4	5	4	22	10	12	15	12	66	
$f_6(x)$	3	5	4	5	5	22	10	12	15	15	66	$\alpha = 3$
	3.5	12	7	7	9	25	24	21	21	27	75	
	3.25	8	4	5	6	23	16	12	15	18	69	
	3.15	7	4	4	5	22	14	12	12	15	66	
	3.35	10	5	5	7	23	20	15	15	21	69	
	3.45	11	5	6	8	23	22	15	18	24	69	

表 2 迭代法 H4、H5、H6 和 H7 与牛顿迭代法的数值结果

$f(x)$	$x_0$	N					NOFE					Root
		NM	H4	H5	H6	H7	NM	H4	H5	H6	H7	
$f_1(x)$	-0.5	112	17	136	21	111	224	51	408	63	333	$\alpha = 1.365\,230\,013\,414\,10$
	1	5	3	5	3	5	10	9	15	9	15	
	1.5	4	3	4	3	4	8	9	12	9	12	
	2	5	3	5	4	5	10	9	15	12	15	
$f_2(x)$	-0.3	54	56	73	7	105	108	168	219	21	315	$\alpha = 0.257\,530\,285\,439\,86$
	2	5	4	5	4	5	10	12	15	12	15	
	3	6	4	6	4	5	12	12	18	12	15	
	0.5	4	3	4	3	4	8	9	12	9	12	
	1	4	3	4	3	4	8	9	12	9	12	
	0.25	3	2	3	2	3	6	6	9	6	9	
	0.1	4	3	4	3	4	8	9	12	9	12	
$f_3(x)$	1	4	3	4	3	4	8	9	12	9	12	$\alpha = 0.739\,085\,133\,215\,16$
	0.5	4	3	4	3	4	8	9	12	9	12	
	1.7	4	3	4	3	4	8	9	12	9	12	
$f_4(x)$	-0.3	5	4	5	4	5	10	12	15	12	15	$\alpha = 2$
	3.5	7	4	7	5	5	14	12	21	15	15	
	2.5	6	4	5	4	4	12	12	15	12	12	
	1.5	7	4	6	24	6	14	12	18	72	18	
	1.7	6	3	5	4	5	12	9	15	12	15	
$f_5(x)$	1.5	6	4	6	5	6	12	12	18	15	18	$\alpha = 2.154\,434\,690\,031\,88$
	2	4	4	5	4	5	8	12	15	12	15	
	2.5	5	4	5	4	5	10	12	15	12	15	
	3	5	4	6	5	5	10	12	18	15	15	
$f_6(x)$	3.5	12	7	12	8	7	24	21	36	24	21	$\alpha = 3$
	3.25	8	5	8	6	7	16	15	24	18	21	
	3.15	7	4	6	5	6	14	12	18	15	18	
	3.35	10	6	9	7	6	20	18	27	21	18	
	3.45	11	6	10	8	7	22	18	30	24	21	

3 结论

本文构造了一类含参数的条件最优的两步迭代方法,其阶数可达到四阶并且符合 Kung-Traub 猜想( $n=3$  情形). 该方法包含了一些已有的方法,并且 Jarratt 方法为本文方法的一种特殊情形. 数值模拟结果表明,本文提出的迭代方法具有很好的可行性.

参考文献:

[1] Ostrowski A M. Solutions of Equations and System of Equations[M]. New York: Academic Press, 1960.

[2] Jarratt P. Some fourth order multipoint iterative methods for solving equations[J]. Math Comput, 1966,20(95): 434-437.

[3] Kung H T, Traub J F. Optimal order of one-point and multipoint iterations[J]. J Assoc Comput Math, 1974,21 (4):643-651.

[4] Frontini M, Sormani E. Third-order methods from quadrature formulae for solving systems of nonlinear equations [J]. Appl Math Comput, 2004,149(3):771-782.

[5] Darvishi M T, Barati A. A third-order Newton-type method to solve systems of nonlinear equations[J]. Appl Math Comput, 2007,187(2):630-635.

[6] Kou J, Li Y, Wang X. A composite fourth-order iterative method for solving non-linear equations[J]. Appl Math Comput, 2007,184(2):471-475.

参考文献：

[1] Yang L Y, Xie X D, Chen F D, et al. Permanence of the periodic predator-prey-mutualist system[J]. Adv Differ Equ, 2015,2015:331. <https://doi.org/10.1186/s13662-015-0654-9>.

[2] 陈兰荪,宋新宇,陆征一. 数学生态学模型与研究方法[M]. 成都:四川科学技术出版社,2003:1.

[3] Chen F D. Permanence for the discrete mutualism model with time delays[J]. Math Comput Model, 2008,47(3/4):431-435.

[4] 余胜斌. 一类离散非自治竞争系统的绝灭性和稳定性[J]. 延边大学学报(自然科学版),2015,41(4):279-284.

[5] Li Y K, Zhang T W. Permanence and almost periodic sequence solution for a discrete delay logistic equation with feedback control[J]. Nonlinear Anal RWA, 2011,12(3):1850-1864.

[6] Yang X T, Liu Y Q, Chen J. Uniform persistence for a discrete predator-prey system with delays[J]. Appl Math Comput, 2011,218(4):1174-1179.

[7] Han R Y, Xie X D, Chen F D. Permanence and global attractivity of a discrete pollination mutualism in plant-pollinator system with feedback controls[J]. Adv Differ Equ, 2016,2016:199. <https://doi.org/10.1186/s13662-016-0889-0>.

[8] Yang W S, Li X P. Permanence of a discrete nonlinear  $n$  species cooperation system with time delays and feedback controls[J]. Appl Math Compu, 2011,218(7):3581-3586.

.....

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[7] Chun C B. Some third-order families of iterative methods for solving nonlinear equations[J]. Appl Math Comput, 2007,188(1):924-933.

[8] Liu X L, Wang X R. Modifications of higher-order convergence for solving nonlinear equations [J]. J Comput Appl Math, 2011,235(17):5105-5111.

[9] Liu X L, Wang X R. A convergence improvement factor and higher-order methods for solving nonlinear equations [J]. Appl Math Comput, 2012,218(15):7871-7875.

[10] Weerakoon S, Fernando G I. A variant of Newton's method with accelerated third-order convergence[J]. Appl Math Lett, 2000,13(8):87-93.

[11] Chun C B. Iterative methods improving Newton's method by the decomposition method[J]. Comput Math Appl, 2005,50(10/12):1559-1568.

[12] Chun C B. A new iterative method for solving nonlinear equations[J]. Appl Math Comput, 2006,178(2):415-422.

[13] Noor M A, Noor K I. Modified iterative methods with cubic convergence for solving nonlinear equations[J]. Appl Math Comput, 2007,184(2):322-325.