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基于汇率和违约双重风险下的外国股票亚式交换期权的定价公式

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摘要: 基于汇率风险暴露下,考虑标的资产价格与该资产所属企业的企业价值以及汇率均遵循对数正态过程的假设下,研究如何把一类外国股票期权推广到股票分红和债务随机时的情况,并利用结构方法推导出以国内股票价格为执行价的该股票亚式交换期权的信用风险定价公式.

关键词: 交换期权; 随机汇率; 信用风险; 鞅测度; 随机债务

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The pricing formulas of a foreign stock asian exchange option with the risk of the exchange rate and the default

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Abstract: Based on the exchange rate risk exposure, on the hypothesis of underlying asset price, enterprise value and exchange rate following logarithmic normal processes, we researched the method of how to expand the foreign stock options to the stock dividend one with stochastic liabilities. By applying the method of structural approach, we derived the pricing formulas of the foreign stock asian exchange options by the strike price with inland stockprice.

Keywords: exchange option; stochastic exchange rate; credit risk; Martingale measure; stochastic liabilities

随着我国综合国力和国际地位的不断提升,市场自由化和产品多样化已经成为了我国金融改革的目标和方向. 由于激烈的竞争和金融产品无专利的原因,使得金融机构不得不不断设计和发展新的风险管理金融产品,其中许多产品是为了迎合顾客特殊需要而设计的. 与汇率相关的交换期权是金融市场上的一种新型交叉货币型期权[1],交换期权给予持有者用一种风险资产交换另一种风险资产的权利,但并不负有义务. 目前,全球经济形势处于动荡阶段,经济复苏乏力,加剧了国际投资人的汇率风险和企业破产风险. 近年来,很多学者研究了外汇型衍生产品,但大多是对单资产的研究[2],或者从微分方程的角度去讨论[3],所采用的执行价一般是用外币和内币进行分析[4]. 本文在考虑股票分红和债务随机时的情况下,利用风险中性方法[5]推导出以国内股票价格为执行价的一类外国股票亚式交换期权的信用风险定价公式.

1　经济变量的随机过程

考虑某个完备概率空间 $(\Omega, F, (F_t)_{t\geqslant 0}, P)$ 上的标准 Brown 运动 W_t, 其中 $(F_t)_{t\geqslant 0}$ 为其上的对 t 递

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增的σ-域. 若以 $S_1(t)$ 表示 t 时以外币计量的外国股票的价格，以 $V(t)$ 表示 t 时以外币计量的外国股票所属企业的企业价值，以 $D(t)$ 表示以外币计量的外国股票所属企业的企业债务，以 $S_2(t)$ 表示 t 时以国内货币(简称内币)计量的国内股票的价格，以 $F(t)$ 表示 t 时以内币计量的一单位外币的价格即 t 时即期汇率，并且假设 $S_1(t),V(t),D(t),S_2(t)$ 和 $F(t)$ 都遵循几何 Brown 运动：

$$\frac{\mathrm{d}S_1(t)}{S_1(t)}=(u_{S_1}-q_{\mathrm{f}})\,\mathrm{d}t+\sigma_{S_1}\mathrm{d}w_t^{S_1},\ \frac{\mathrm{d}V(t)}{V(t)}=u_V\mathrm{d}t+\sigma_V\mathrm{d}w_t^V,$$

$$\frac{\mathrm{d}D(t)}{D(t)}=u_D\mathrm{d}t+\sigma_D\mathrm{d}w_t^D,\ \frac{\mathrm{d}S_2(t)}{S_2(t)}=(u_{S_2}-q_{\mathrm{d}})\,\mathrm{d}t+\sigma_{S_2}\mathrm{d}w_t^{S_2},\ \frac{\mathrm{d}F(t)}{F(t)}=u_F\mathrm{d}t+\sigma_F\mathrm{d}w_t^F. \tag{1}$$

其中：初始条件 $S_1(t),V(t),D(t),S_2(t)$ 和 $F(t)$ 均为 F_t 可测，且 $0\leqslant t\leqslant T$；参数 $u_{S_1},u_V,u_D,u_{S_2},u_F$ 和 $\sigma_{S_1},\sigma_V,\sigma_D,\sigma_{S_2},\sigma_F$ 分别表示期望收益率和波动率；q_{f} 和 q_{d} 分别为 $S_1(t)$ 和 $S_2(t)$ 的红利率常数；$w_t^{S_1},w_t^V$, $w_t^D,w_t^{S_2}$ 和 w_t^F 均是测度 P 下的标准 Wiener 过程. 设国外和国内无风险利率常数分别为 r_{f} 和 r_{d}，用 ρ_{ij} 表示 w_t^i 和 w_t^j 的相关系数，即 $\mathrm{cov}(\mathrm{d}w_t^i,\mathrm{d}w_t^j)=\rho_{ij}\mathrm{d}t,\ i,j\in\{S_1,V,D,S_2,F\}$.

2　风险中性概率测度和数学模型

2.1　风险中性概率测度

根据期权定价的基本理论[6]可知，当 $S_1(t),V(t),D(t),S_2(t)$ 和 $F(t)$ 的预期增长率分别变换成 $\bar{\alpha}_{S_1}=r_{\mathrm{f}}-q_{\mathrm{f}}-\rho_{S_1F}\sigma_{S_1}\sigma_F$, $\bar{\alpha}_V=r_{\mathrm{f}}-\rho_{VF}\sigma_V\sigma_F$, $\bar{\alpha}_D=r_{\mathrm{f}}-\rho_{DF}\sigma_D\sigma_F$, $\bar{\alpha}_{S_2}=r_{\mathrm{d}}$, $\bar{\alpha}_F=r_{\mathrm{d}}-r_{\mathrm{f}}$ 时，可以得到衍生产品在风险中性测度下的正确估值. 引入一个与测度 P 等价的鞅测度 Q[7]：

$$\frac{\mathrm{d}Q}{\mathrm{d}P}=\exp\left\{rw-\frac{1}{2}\,|r|^2t\right\},$$

其中：$r=(r_{S_1},r_V,r_D,r_{S_2},r_F)^{\mathrm{T}}$；$r_i=\dfrac{\bar{\alpha}_i-u_i}{\sigma_i}$, $i\in\{S_1,V,D,S_2,F\}$；$|r|$ 表示 r 的模，$w=(w_{S_1},w_V,w_D,w_{S_2},w_F)^{\mathrm{T}}$. 由 Girsanov’s 定理知，在鞅测度 Q 下，式(1)可转化为：

$$\frac{\mathrm{d}S_1(t)}{S_1(t)}=\bar{\alpha}_{S_1}\mathrm{d}t+\sigma_{S_1}\mathrm{d}\widetilde{w}_t^{S_1},\ \frac{\mathrm{d}V(t)}{V(t)}=\bar{\alpha}_V\mathrm{d}t+\sigma_V\mathrm{d}\widetilde{w}_t^V,$$

$$\frac{\mathrm{d}D(t)}{D(t)}=\bar{\alpha}_D\mathrm{d}t+\sigma_D\mathrm{d}\widetilde{w}_t^D,\ \frac{\mathrm{d}S_2(t)}{S_2(t)}=\bar{\alpha}_{S_2}\mathrm{d}t+\sigma_{S_2}\mathrm{d}\widetilde{w}_t^{S_2},\ \frac{\mathrm{d}F(t)}{F(t)}=\bar{\alpha}_F\mathrm{d}t+\sigma_F\mathrm{d}\widetilde{w}_t^F. \tag{2}$$

其中 $\widetilde{w}_t^i\ (i\in\{S_1,V,D,S_2,F\})$ 为测度 Q 下的标准 Wiener 过程，且 $\mathrm{cov}(\mathrm{d}\widetilde{w}_t^i,\mathrm{d}\widetilde{w}_t^j)=\rho_{ij}\mathrm{d}t,\ i,j\in\{S_1,V,D,S_2,F\}$.

2.2　几何平均亚式交换期权的定义和数学模型

设 $G_i(T)\ (i\in\{S_1,V,S_2,F\})$ 分别是 $S_1(t),V(t),S_2(t)$ 和 $F(t)$ 在时间段 $[T_0,T]$ 上的离散几何平均值[8]，用 ρ_{ij} 分别表示 w_t^i 和 $w_t^j\ (i,j\in\{G_{S_1},G_V,G_{S_2},G_F\})$ 的相关系数，用 $t_k=T_0+k\Delta T\ (k=0,1,2,\cdots,n)$ 表示对区间 $[T_0,T]$ 的分割，$\Delta T=\dfrac{T-T_0}{n}$, $t_n=T$，于是有

$$G_{S_1}(T)=S_1(t)\exp\left\{\frac{\bar{\alpha}_{S_1}-\frac{\sigma_{S_1}^2}{2}}{n}\cdot\sum_{k=1}^{n}(t_k-t)+\frac{\sigma_{S_1}}{n}\sum_{j=1}^{n}(\widetilde{w}_{t_j}^{S_1}-\widetilde{w}_t^{S_1})\right\}=S_1(t)\exp\left\{\frac{\bar{\alpha}_{S_1}-\frac{\sigma_{S_1}^2}{2}}{n}\cdot\right.$$

$$\left.\sum_{k=1}^{n}(t_k-t)+\frac{\sigma_{S_1}}{n}\left[n(\widetilde{w}_{t_1}^{S_1}-\widetilde{w}_t^{S_1})+(n-1)\,(\widetilde{w}_{t_2}^{S_1}-\widetilde{w}_{t_1}^{S_1})+\cdots+(\widetilde{w}_{t_n}^{S_1}-\widetilde{w}_{t_{n-1}}^{S_1})\right]\right\}\triangleq S_1(t)\mathrm{e}^{\xi_1}.$$

因为 $\widetilde{w}_{t_1}^{S_1}-\widetilde{w}_t^{S_1},\widetilde{w}_{t_k}^{S_1}-\widetilde{w}_{t_{k-1}}^{S_1}\ (k=2,3,\cdots,n)$ 之间是相互独立的正态随机变量[9]，故 ξ_1 服从正态分布，且均值和方差分别为：

$$E(\xi_1)=\left(\bar{\alpha}_{S_1}-\frac{\sigma_{S_1}^2}{2}\right)\left[(T-t)-\frac{n-1}{2n}(T-T_0)\right]\triangleq\lambda_1;$$

$$\operatorname{var}(\xi_1)=\sigma_{S_1}^2\left[(T-t)-\frac{4n^2-3n-1}{6n}(T-T_0)\right]\triangleq\theta_1^2.$$

同理,设$\sigma_{G_i}^2$和μ_{G_i}分别为$G_i(T)$ $(i\in\{S_1,V,S_2,F\})$的方差和期望,$\tau=T-t$,得:$\sigma_{G_i}^2\tau=\lambda_m$,$\left(\mu_{G_i}-\frac{\sigma_{G_i}^2}{2}\right)\tau=\theta_m^2$,$i\in\{S_1,S_2,V,F\}$,$m\in\{1,2,3,4\}$.设$\eta_m=\frac{\xi_m-\lambda_m}{\theta_m}$,$m\in\{1,2,3,4\}$,有$\eta_m\sim N(0,1)$,所以$G_i(T)=i(t)\exp(\lambda_m+\theta_m\eta_m)$,$i\in\{S_1,S_2,V,F\}$,$m\in\{1,2,3,4\}$.

考虑用一种风险资产去交换另一种风险资产的交换期权的定价问题.普通交换期权的到期收益函数为$\max(S_1(T)-S_2(T),0)$(用资产2交换资产1)[10],而随机汇率下的外国股票几何平均亚式交换期权的到期收益可定义为$\max(G_{S_1}(T)G_F(T)-G_{S_2}(T),0)$(以国内股票$S_2$的几何平均为执行价).假设期权的允诺支付为$X(T)$,$\delta_T$为支付比($0\leqslant\delta_T<1$),则破产(破产是规定在$T$时刻才有可能发生的)后的支付为$X^d(T)=X(T)\delta_T$[11].设$\delta_T=V(T)/D(T)$,其中$D(T)$为$T$时全部债务,$V(T)$为$T$时全部资产.本研究所讨论的企业价值取为$G_V(T)$($V_t$在时间段$[T_0,T]$上的离散几何平均值),故$\delta_T=G_V(T)/D(T)$,$\delta_t=G_V(t)/D(t)$,其中$G_V(t)=\mathrm{e}^{-r\tau}G_V(T)$,$D(t)=\mathrm{e}^{-r\tau}D(T)$.于是有:

$$\delta_T=\delta_t\cdot\exp\left[\left(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{G_V}^2\right)\tau+\sigma_\delta(\widetilde{w}_T^\delta-\widetilde{w}_t^\delta)\right],$$

$$\sigma_\delta=\sqrt{\sigma_{G_V}^2+\sigma_D^2-2\rho_{G_VD}\,\sigma_{G_V}\sigma_D},$$

其中ρ_{G_iM}表示G_i和M的相关系数,M可取D或者G_i $(i\in\{S_1,V,S_2,F\})$.

3 多元标准正态分布函数和多变量转移密度函数的有关说明

根据参考文献[12],设$n+1$元标准正态分布函数为$N_{n+1}(a_0,a_1,a_2,\cdots,a_n;\boldsymbol{\Sigma}_{n+1})$,则

$$N_{n+1}(a_0,\cdots,a_n;\boldsymbol{\Sigma}_{n+1})=\int_{-\infty}^{a_n}\cdots\int_{-\infty}^{a_0}\frac{1}{(2\pi)^{(n+1)/2}\sqrt{|\boldsymbol{\Sigma}_{n+1}|}}\cdot$$

$$\exp[(x_0,\cdots,x_n)\boldsymbol{\Sigma}_{n+1}(x_0,\cdots,x_n)^{\mathrm{T}}]\,\mathrm{d}x_0\cdots\mathrm{d}x_n.$$

其中:$\boldsymbol{\Sigma}_{n+1}=\begin{pmatrix}1&\rho_{01}&\cdots&\rho_{0n}\\\rho_{01}&1&\cdots&\rho_{1n}\\\vdots&\vdots&\ddots&\vdots\\\rho_{0n}&\rho_{1n}&\cdots&1\end{pmatrix}_{(n+1)\times(n+1)}$,$\rho_{ij}$是变量$x_i$与$x_j$的相关系数,$i,j=0,1,\cdots,n$.

设$n+1$个随机变量$S_T,V_{T_1},\cdots,V_{T_n}$的转移密度函数为$\varphi(S_T,V_{T_1},\cdots,V_{T_n};S_t,V_1,\cdots,V_n)$,则有

$$\varphi(S_T,V_{T_1},\cdots,V_{T_n};S_t,V_1,\cdots,V_n)=\frac{1}{(2\pi\tau)^{(n+1)/2}\sqrt{|\boldsymbol{\Sigma}_{n+1}|}\sigma_0S_T\prod_{i=1}^n\sigma_0V_{T_i}}\cdot$$

$$\exp\left[-\frac{1}{2}(x_0,x_1,\cdots,x_n)\boldsymbol{\Sigma}_{n+1}^{-1}(x_0,x_1,\cdots,x_n)^{\mathrm{T}}\right],$$

其中$x_0=\left[\ln S_T-\left(\ln S_t+\left(r-\frac{\sigma_0^2}{2}\right)\tau\right)\right]/\sigma_0\sqrt{\tau}$,$x_i=\left[\ln V_{T_i}-\left(\ln V_i+\left(r-\frac{\sigma_i^2}{2}\right)\tau\right)\right]/\sigma_i\sqrt{\tau}$,$i=1,2,\cdots,n$.

4 期权定价公式及其求解

4.1 模型的建立

国际金融市场的动荡多变,在金融投资过程中国际投资人不仅会遇上汇率风险,而且可能会遇到外国公司的违约风险,正因为违约风险的存在,企业到期时的支付就被分成两部分:一部分是没有违约时的支付,另一部分是发生违约时的补偿支付.由此可以得出以国内股票S_2的几何平均为执行价的外国

股票几何平均亚式交换期权的到期收益为:

$$X_T=(G_{S_1}(T)G_F(T)-G_{S_2}(T))^+I_{\{\delta_T\geqslant 1\}}+(G_{S_1}(T)G_F(T)-G_{S_2}(T))^+\delta_T I_{\{\delta_T<1\}}.$$

设 $E\langle X\mid F_t\rangle$ 表示随机变量 X 关于到时间 t 为止可获得的信息的条件期望,根据鞅定价理论[11],期权在 t 时刻的价值为到期收益期望的折现,即:

$$X_t=B_tE_Q[B_T^{-1}(G_{S_1}(T)G_F(T)-G_{S_2}(T))^+(I_{\{\delta_T\geqslant 1\}}+\delta_T I_{\{\delta_T<1\}})\mid F_t],$$

其中 $\delta_T=G_V(T)/D(T)$. 令 $\psi_T=G_{S_1}(T)G_F(T)$, $\phi_T=\psi_T/G_{S_2}(T)$, 则在风险中性测度 Q 下,有

$$\frac{\mathrm{d}\psi(t)}{\psi(t)}=r_\psi\mathrm{d}t+\sigma_\psi\mathrm{d}\widetilde{w}_t^\psi,\ \frac{\mathrm{d}\phi(t)}{\phi(t)}=r_\phi\mathrm{d}t+\sigma_\phi\mathrm{d}\widetilde{w}_t^\phi, \tag{3}$$

其中 $r_\psi=r_\mathrm{d}-q_\mathrm{f}-\rho_{G_{S_1}G_F}\sigma_{G_{S_1}}\sigma_{G_F}$, $\sigma_\psi^2=\sigma_{G_{S_1}}^2+2\rho_{G_{S_1}G_F}\sigma_{G_{S_1}}\sigma_{G_F}+\sigma_{G_F}^2$. 由此式(3) 可变为

$$\phi(T)=\phi(t)\exp\left[\left(\frac{1}{2}\sigma_{G_{S_2}}^2-\frac{1}{2}\sigma_\psi^2\right)\tau+\sigma_\phi(\widetilde{w}_T^\phi-\widetilde{w}_t^\phi)\right].$$

设 ϕ 与 δ 的相关系数为 $\rho_{\phi\delta}$, 则 $\rho_{\phi\delta}=(\rho_{\phi G_V}\sigma_{G_V}-\rho_{\phi D}\sigma_D)/\sigma_\delta$, 其中:

$$\rho_{\phi G_V}=(\rho_{G_{S_1}G_V}\sigma_{G_{S_1}}+\rho_{G_FG_V}\sigma_{G_F}-\rho_{G_{S_2}G_V}\sigma_{G_{S_2}})/\sigma_\phi,\ \rho_{\phi D}=(\rho_{G_{S_1}D}\sigma_{G_{S_1}}+\rho_{G_FD}\sigma_{G_F}-\rho_{G_{S_2}D}\sigma_{G_{S_2}})/\sigma_\phi,$$

$$\sigma_\phi^2=\sigma_{G_{S_1}}^2+\sigma_{G_F}^2+2\rho_{G_{S_1}G_F}\sigma_{G_{S_1}}\sigma_{G_F}+\sigma_{G_{S_2}}^2-2\rho_{G_{S_1}G_{S_2}}\sigma_{G_{S_1}}\sigma_{G_{S_2}}-2\rho_{G_{S_2}G_F}\sigma_{G_{S_2}}\sigma_{G_F}.$$

所以有 $X_t=B_tE_Q\{B_T^{-1}[G_{S_2}(T)(\phi(T)-1)^+][I_{\{\delta_T\geqslant 1\}}+\delta_TI_{\{\delta_T<1\}}]\mid F_t\}$. 令 $X_t=E_1-E_2+E_3-E_4$, 其中:

$$E_1=B_tE_Q[B_T^{-1}G_{S_2}(T)\phi(T)I_{\{\phi(T)>1\}}I_{\{\delta_T\geqslant 1\}}\mid F_t],$$

$$E_2=B_tE_Q[B_T^{-1}G_{S_2}(T)I_{\{\phi(T)>1\}}I_{\{\delta_T\geqslant 1\}}\mid F_t],$$

$$E_3=B_tE_Q[B_T^{-1}G_{S_2}(T)\phi(T)\delta_TI_{\{\phi(T)>1\}}I_{\{\delta_T<1\}}\mid F_t],$$

$$E_4=B_tE_Q[B_T^{-1}G_{S_2}(T)\delta_TI_{\{\phi(T)>1\}}I_{\{\delta_T<1\}}\mid F_t].$$

4.2　模型的求解

假设国内无风险利率 r_d 为常数,则 $B_tE_Q[B_T^{-1}\mid F_t]=\mathrm{e}^{-r_\mathrm{d}\tau}$, 其中 $\tau=T-t$. 为了方便数学符号表示,本文做以下符号更改:

$$G_{S_1}\triangleq G_1,\ G_{S_2}\triangleq G_2,\ G_{S_2}(T)\triangleq G_{2T},\ G_{S_2}(t)\triangleq G_{2t},\ \phi(T)=\phi_T,\ \phi(t)=\phi_t,$$

$$\rho_{\phi\delta}=\frac{\rho_{\phi G_V}\sigma_{G_V}-\rho_{\phi D}\sigma_D}{\sigma_\delta}\triangleq\rho_{01},\ \rho_{G_2\delta}=\frac{\rho_{G_2G_V}\sigma_{G_V}-\rho_{G_2D}\sigma_D}{\sigma_\delta}\triangleq\rho_{12},\ \rho_{G_2\phi}=\frac{\rho_{G_1G_2}\sigma_{G_1}+\rho_{G_2G_F}\sigma_F-\sigma_{G_2}}{\sigma_\phi}\triangleq\rho_{02}.$$

1) E_1 的计算.

$$E_1=B_tE_Q[B_T^{-1}G_{2T}\phi_TI_{\{\phi_T>1\}}I_{\{\delta_T\geqslant 1\}}\mid F_t]=$$

$$\mathrm{e}^{-r_\mathrm{d}\tau}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}G_{2T}\phi_TI_{\{\phi_T>1\}}I_{\{\delta_T\geqslant 1\}}\varphi_3(G_{2T},\phi_T,\delta_T,G_{2t},\phi_t,\delta_t)\,\mathrm{d}G_{2T}\mathrm{d}\phi_T\mathrm{d}\delta_T,$$

其中:

$$\varphi_3(G_{2T},\phi_T,\delta_T,G_{2t},\phi_t,\delta_t)=\frac{1}{(2\pi\tau)^{3/2}\sqrt{|\boldsymbol{\Sigma}_3|}\sigma_{G_2}G_{2T}\sigma_\phi\phi_T\sigma_\delta\delta_T}\cdot$$

$$\exp\left[-\frac{1}{2}(x_0,x_1,x_2)\boldsymbol{\Sigma}_3^{-1}(x_0,x_1,x_2)^\mathrm{T}\right],$$

$$\boldsymbol{\Sigma}_3=\begin{pmatrix}1&\rho_{01}&\rho_{02}\\\rho_{01}&1&\rho_{12}\\\rho_{02}&\rho_{12}&1\end{pmatrix},\ \boldsymbol{\Sigma}_3^{-1}=\frac{1}{|\boldsymbol{\Sigma}_3|}\begin{pmatrix}a_{00}&a_{01}&a_{02}\\a_{01}&a_{11}&a_{12}\\a_{02}&a_{12}&a_{22}\end{pmatrix},\ |\boldsymbol{\Sigma}_3|=1-\rho_{01}^2-\rho_{02}^2-\rho_{12}^2+2\rho_{01}\rho_{02}\rho_{12},$$

$$a_{00}=1-\rho_{12}^2,\ a_{11}=1-\rho_{02}^2,\ a_{22}=1-\rho_{01}^2,\ a_{01}=\rho_{02}\rho_{12}-\rho_{01},\ a_{02}=\rho_{01}\rho_{12}-\rho_{02},\ a_{12}=\rho_{01}\rho_{02}-\rho_{12}.$$

由示性函数性质可得,存在 a_0^0 和 a_1^0,满足:

$$\phi_T=\phi_t\cdot\exp\left[\left(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2\right)\tau+\sigma_\phi\sqrt{\tau}(-a_0^0)\right]=1,$$

$$\delta_T=\delta_t\cdot\exp\left[\left(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{G_V}^2\right)\tau+\sigma_\delta\sqrt{\tau}\,(-a_1^0)\right]=1.$$

由此解得：$a_0^0=\dfrac{\ln\phi_t+\left(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2\right)\tau}{\sigma_\phi\sqrt{\tau}}$，$a_1^0=\dfrac{\ln\delta_t+\left(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{G_V}^2\right)\tau}{\sigma_\delta\sqrt{\tau}}$，即：

$$E_1=\mathrm{e}^{-r_\mathrm{d}\tau}\int_{-\infty}^{+\infty}\int_{-a_1^0}^{+\infty}\int_{-a_0^0}^{+\infty}S_{2t}\cdot\exp(\lambda_2+\theta_2x_2)\,\phi_t\exp\left[\left(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2\right)\tau+\sigma_\phi\sqrt{\tau}\,x_0\right]\frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{|\boldsymbol{\Sigma}_3|}}\cdot$$
$$\exp[-\frac{1}{2|\boldsymbol{\Sigma}_3|}(a_{00}x_0^2+a_{11}x_1^2+a_{22}x_2^2+2a_{01}x_0x_1+2a_{02}x_0x_2+2a_{12}x_1x_2)]\mathrm{d}x_0\mathrm{d}x_1\mathrm{d}x_2.$$

设存在 a,b,c,w_1 使得

$$w_1+\theta_2x_2+\sigma_\phi\sqrt{\tau}\,x_0-\frac{1}{2|\boldsymbol{\Sigma}_3|}(a_{00}x_0^2+a_{11}x_1^2+a_{22}x_2^2+2a_{01}x_0x_1+2a_{02}x_0x_2+2a_{12}x_1x_2)=$$
$$-\frac{1}{2|\boldsymbol{\Sigma}_3|}[a_{00}(x_0-a)^2+a_{11}(x_1-b)^2+a_{22}(x_2-c)^2+2a_{01}(x_0-a)(x_1-b)+$$
$$2a_{02}(x_0-a)(x_2-c)+2a_{12}(x_1-b)(x_2-c)].$$

利用比较系数法，得：

$$a_{00}a^2+a_{11}b^2+a_{22}c^2+2a_{01}ab+2a_{02}ac+2a_{12}bc=w_1(-2|\boldsymbol{\Sigma}_3|)\,,\qquad(*)$$
$$aa_{00}+ba_{01}+ca_{02}=\sigma_\phi\sqrt{\tau}\,|\boldsymbol{\Sigma}_3|,\ aa_{01}+ba_{11}+ca_{12}=0,\ aa_{02}+ba_{12}+ca_{22}=\theta_2|\boldsymbol{\Sigma}_3|.$$

根据克莱姆法则[13]，解得：

$$a=\begin{vmatrix}\sigma_\phi\sqrt{\tau}\,|\boldsymbol{\Sigma}_3| & a_{01} & a_{02}\\ 0 & a_{11} & a_{12}\\ \theta_2|\boldsymbol{\Sigma}_3| & a_{12} & a_{22}\end{vmatrix}\Bigg/\begin{vmatrix}a_{00} & a_{01} & a_{02}\\ a_{01} & a_{11} & a_{12}\\ a_{02} & a_{12} & a_{22}\end{vmatrix}=\frac{\sigma_\phi\sqrt{\tau}\cdot\begin{vmatrix}a_{11} & a_{12}\\ a_{12} & a_{22}\end{vmatrix}+\theta_2\cdot\begin{vmatrix}a_{01} & a_{02}\\ a_{11} & a_{12}\end{vmatrix}}{|\boldsymbol{\Sigma}_3|}=$$
$$\sigma_\phi\sqrt{\tau}+\theta_2\rho_{02},$$

$$b=\begin{vmatrix}a_{00} & \sigma_\phi\sqrt{\tau}\,|\boldsymbol{\Sigma}_3| & a_{02}\\ a_{01} & 0 & a_{12}\\ a_{02} & \theta_2|\boldsymbol{\Sigma}_3| & a_{22}\end{vmatrix}\Bigg/\begin{vmatrix}a_{00} & a_{01} & a_{02}\\ a_{01} & a_{11} & a_{12}\\ a_{02} & a_{12} & a_{22}\end{vmatrix}=\frac{\sigma_\phi\sqrt{\tau}\cdot\begin{vmatrix}a_{01} & a_{12}\\ a_{02} & a_{22}\end{vmatrix}+\theta_2\cdot\begin{vmatrix}a_{00} & a_{02}\\ a_{01} & a_{12}\end{vmatrix}}{|\boldsymbol{\Sigma}_3|}=$$
$$\sigma_\phi\sqrt{\tau}\,\rho_{01}+\theta_2\rho_{12},$$

$$c=\begin{vmatrix}a_{00} & a_{01} & \sigma_\phi\sqrt{\tau}\,|\boldsymbol{\Sigma}_3|\\ a_{01} & a_{11} & 0\\ a_{02} & a_{12} & \theta_2|\boldsymbol{\Sigma}_3|\end{vmatrix}\Bigg/\begin{vmatrix}a_{00} & a_{01} & a_{02}\\ a_{01} & a_{11} & a_{12}\\ a_{02} & a_{12} & a_{22}\end{vmatrix}=\frac{\sigma_\phi\sqrt{\tau}\cdot\begin{vmatrix}a_{01} & a_{11}\\ a_{02} & a_{12}\end{vmatrix}+\theta_2\cdot\begin{vmatrix}a_{00} & a_{01}\\ a_{01} & a_{11}\end{vmatrix}}{|\boldsymbol{\Sigma}_3|}=$$
$$\sigma_\phi\sqrt{\tau}\,\rho_{02}+\theta_2.$$

把上述 a,b,c 代入式(*)可得 $w_1=-\dfrac{1}{2}(\sigma_\phi^2\tau+\theta_2^2)-\sigma_\phi\sqrt{\tau}\,\rho_{02}\theta_2$，因此可以令：

$$\bar{x}_0=x_0-(\sigma_\phi\sqrt{\tau}+\theta_2\rho_{02}),\ \bar{x}_1=x_1-(\sigma_\phi\sqrt{\tau}\,\rho_{01}+\theta_2\rho_{12}),\ \bar{x}_2=x_2-(\sigma_\phi\sqrt{\tau}\,\rho_{02}+\theta_2).$$

相应的，有 $a_0^1=a_0^0+(\sigma_\phi\sqrt{\tau}+\theta_2\rho_{02})$，$a_1^1=a_1^0+(\sigma_\phi\sqrt{\tau}\,\rho_{01}+\theta_2\rho_{12})$，所以

$$E_1=S_{2t}\,\phi_t\exp\left[-r_\mathrm{d}\tau+\lambda_2+\left(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2\right)\tau-w_1\right]\int_{-\infty}^{+\infty}\int_{-a_1^1}^{+\infty}\int_{-a_0^1}^{+\infty}\frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{|\boldsymbol{\Sigma}_3|}}\cdot$$
$$\exp\left[-\frac{1}{2|\boldsymbol{\Sigma}_3|}(a_{00}\bar{x}_0^2+a_{11}\bar{x}_1^2+a_{22}\bar{x}_2^2+2a_{01}\bar{x}_0\bar{x}_1+2a_{02}\bar{x}_0\bar{x}_2+2a_{12}\bar{x}_1\bar{x}_2)\right]\mathrm{d}\bar{x}_0\mathrm{d}\bar{x}_1\mathrm{d}\bar{x}_2=$$
$$S_{2t}\,\phi_t\exp(m_1)\,N_2(a_0^1,a_1^1;\boldsymbol{\Sigma}_2)\,,$$

其中 $w_1=-\frac{1}{2}(\sigma_\phi^2\tau+\theta_2^2)-\sigma_\phi\sqrt{\tau}\rho_{02}\theta_2$, $m_1=-r_d\tau+\lambda_2+\left(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2\right)\tau-w_1$, $a_0^1=a_0^0+(\sigma_\phi\sqrt{\tau}+\theta_2\rho_{02})$, $a_1^1=a_1^0+(\sigma_\phi\sqrt{\tau}\rho_{01}+\theta_2\rho_{12})$, $\boldsymbol{\Sigma}_2=\begin{pmatrix}1 & \rho_{01}\\ \rho_{01} & 1\end{pmatrix}$.

2) E_2 的计算.类似 E_1 的计算过程,令 $\hat{x}_0=x_0-\theta_2\rho_{02}$, $\hat{x}_1=x_1-\theta_2\rho_{12}$, $\hat{x}_2=x_2-\theta_2$.相应的,有 $a_0^2=a_0^0+\theta_2\rho_{02}$, $a_1^2=a_1^0+\theta_2\rho_{12}$,所以

$$E_2=S_{2t}\exp(-r_d\tau+\lambda_2-w_2)\int_{-\infty}^{+\infty}\int_{-a_1^2}^{+\infty}\int_{-a_0^2}^{+\infty}\frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{|\boldsymbol{\Sigma}_3|}}\cdot\exp[-\frac{1}{2|\boldsymbol{\Sigma}_3|}(a_{00}\hat{x}_0^2+a_{11}\hat{x}_1^2+a_{22}\hat{x}_2^2+2a_{01}\hat{x}_0\hat{x}_1+2a_{02}\hat{x}_0\hat{x}_2+2a_{12}\hat{x}_1\hat{x}_2)]\mathrm{d}\hat{x}_0\mathrm{d}\hat{x}_1\mathrm{d}\hat{x}_2=S_{2t}\exp(m_2)N_2(a_0^2,a_1^2;\boldsymbol{\Sigma}_2),$$

其中 $w_2=-\frac{\theta_2^2}{2}$, $m_2=-r_d\tau+\lambda_2-w_2$, $a_0^2=a_0^0+\theta_2\rho_{02}$, $a_1^2=a_1^0+\theta_2\rho_{12}$.

3) E_3 的计算.类似 E_1 的计算过程,令 $\tilde{x}_0=x_0-(\sigma_\phi\sqrt{\tau}+\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $\tilde{x}_1=x_1-(\sigma_\phi\sqrt{\tau}\rho_{01}+\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$, $\tilde{x}_2=x_2-(\sigma_\phi\sqrt{\tau}\rho_{02}+\sigma_\delta\sqrt{\tau}\rho_{12}+\theta_2)$.相应的,有 $a_0^3=a_0^0+(\sigma_\phi\sqrt{\tau}+\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $a_1^3=a_1^0+(\sigma_\phi\sqrt{\tau}\rho_{01}+\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$,所以

$$E_3=S_{2t}\phi_t\delta_t\exp\left[-r_d\tau+\lambda_2+\left(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2\right)\tau+\left(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{GV}^2\right)\tau-w_3\right]\cdot\int_{-\infty}^{+\infty}\int_{-a_1^3}^{+\infty}\int_{-a_0^3}^{+\infty}\frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{|\boldsymbol{\Sigma}_3|}}\exp[-\frac{1}{2|\boldsymbol{\Sigma}_3|}(a_{00}\tilde{x}_0^2+a_{11}\tilde{x}_1^2+a_{22}\tilde{x}_2^2+2a_{01}\tilde{x}_0\tilde{x}_1+2a_{02}\tilde{x}_0\tilde{x}_2+2a_{12}\tilde{x}_1\tilde{x}_2)]\mathrm{d}\tilde{x}_0\mathrm{d}\tilde{x}_1\mathrm{d}\tilde{x}_2=S_{2t}\phi_t\delta_t\exp(m_3)N_2(a_0^3,a_1^3;\boldsymbol{\Sigma}_2),$$

其中:$w_3=-\frac{1}{2}(\sigma_\phi^2\tau+\sigma_\delta^2\tau+2\sigma_\phi\sigma_\delta\tau\rho_{01}+\theta_2^2)-(\sigma_\phi\sqrt{\tau}\rho_{02}+\sigma_\delta\sqrt{\tau}\rho_{12})\theta_2$, $m_3=-r_d\tau+\lambda_2+(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2)\tau+(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{GV}^2)\tau-w_3$, $a_0^3=a_0^0+(\sigma_\phi\sqrt{\tau}+\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $a_1^3=a_1^0+(\sigma_\phi\sqrt{\tau}\rho_{01}+\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$.

4) E_4 的计算.类似 E_1 的计算过程,令 $\hat{x}_0=x_0-(\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $\hat{x}_1=x_1-(\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$, $\hat{x}_2=x_2-(\sigma_\delta\sqrt{\tau}\rho_{12}+\theta_2)$.相应的,有 $a_0^4=a_0^0+(\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $a_1^4=a_1^0+(\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$,所以

$$E_4=S_{2t}\delta_t\exp\left[-r_d\tau+\lambda_2+\left(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{GV}^2\right)\tau-w_4\right]\int_{-\infty}^{+\infty}\int_{-a_1^4}^{+\infty}\int_{-a_0^4}^{+\infty}\frac{1}{(2\pi)^{\frac{3}{2}}\sqrt{|\boldsymbol{\Sigma}_3|}}\cdot\exp\left[-\frac{1}{2|\boldsymbol{\Sigma}_3|}(a_{00}\hat{x}_0^2+a_{11}\hat{x}_1^2+a_{22}\hat{x}_2^2+2a_{01}\hat{x}_0\hat{x}_1+2a_{02}\hat{x}_0\hat{x}_2+2a_{12}\hat{x}_1\hat{x}_2)\right]\mathrm{d}\hat{x}_0\mathrm{d}\hat{x}_1\mathrm{d}\hat{x}_2=S_{2t}\delta_t\exp(m_4)N_2(a_0^4,a_1^4;\boldsymbol{\Sigma}_2),$$

其中 $w_4=-\frac{1}{2}(\sigma_\delta^2\tau+\theta_2^2)-\sigma_\delta\sqrt{\tau}\rho_{12}\theta_2$, $m_4=-r_d\tau+\lambda_2+(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{GV}^2)\tau-w_4$, $a_0^4=a_0^0+(\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $a_1^4=a_1^0+(\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$.

4.3　模型结论及展望

综合上述,在考虑股票分红和债务随机时的情况下,以国内股票价格为执行价的该股票亚式交换期权的信用风险定价公式为:

$$X_t=S_{2t}\exp(m_2)N_2(a_0^2,a_1^2;\boldsymbol{\Sigma}_2)-S_{2t}\exp(m_2)N_2(a_0^2,a_1^2;\boldsymbol{\Sigma}_2)+S_{2t}\phi_t\delta_t\exp(m_3)N_2(a_0^3,a_1^3;\boldsymbol{\Sigma}_2)-S_{2t}\delta_t\exp(m_4)N_2(a_0^4,a_1^4;\boldsymbol{\Sigma}_2),$$

其中 $a_0^0=\dfrac{\ln\phi_t+(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2)\tau}{\sigma_\phi\sqrt{\tau}}$, $a_1^0=\dfrac{\ln\delta_t+(\frac{1}{2}\sigma_D^2-\frac{1}{2}\sigma_{GV}^2)\tau}{\sigma_\delta\sqrt{\tau}}$, $w_1=-\frac{1}{2}(\sigma_\phi^2\tau+\theta_2^2)-\sigma_\phi\sqrt{\tau}\rho_{02}\theta_2$,

$m_1=-r_d\tau+\lambda_2+(\frac{1}{2}\sigma_{G_2}^2-\frac{1}{2}\sigma_\psi^2)\tau-w_1$, $a_0^1=a_0^0+\left(\sigma_\phi\sqrt{\tau}+\theta_2\rho_{02}\right)$, $a_1^1=a_1^0+\left(\sigma_\phi\sqrt{\tau}\rho_{01}+\theta_2\rho_{12}\right)$,

$\boldsymbol{\Sigma}_2=\begin{pmatrix}1 & \rho_{01}\\ \rho_{01} & 1\end{pmatrix}$, $w_2=-\dfrac{\theta_2^2}{2}$, $m_2=-r_{\mathrm{d}}\tau+\lambda_2-w_2$, $a_0^2=a_0^0+\theta_2\rho_{02}$, $a_1^2=a_1^0+\theta_2\rho_{12}$, $w_3=-\dfrac{1}{2}(\sigma_\phi^2\tau+\sigma_\delta^2\tau+2\sigma_\phi\sigma_\delta\tau\rho_{01}+\theta_2^2)-(\sigma_\phi\sqrt{\tau}\rho_{02}+\sigma_\delta\sqrt{\tau}\rho_{12})\theta_2$, $m_3=-r_{\mathrm{d}}\tau+\lambda_2+(\dfrac{1}{2}\sigma_{G_2}^2-\dfrac{1}{2}\sigma_\psi^2)\tau+(\dfrac{1}{2}\sigma_D^2-\dfrac{1}{2}\sigma_{GV}^2)\tau-w_3$, $a_0^3=a_0^0+(\sigma_\phi\sqrt{\tau}+\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $a_1^3=a_1^0+(\sigma_\phi\sqrt{\tau}\rho_{01}+\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$, $w_4=-\dfrac{1}{2}(\sigma_\delta^2\tau+\theta_2^2)-\sigma_\delta\sqrt{\tau}\rho_{12}\theta_2$, $m_4=-r_{\mathrm{d}}\tau+\lambda_2+(\dfrac{1}{2}\sigma_D^2-\dfrac{1}{2}\sigma_{GV}^2)\tau-w_4$, $a_0^4=a_0^0+(\sigma_\delta\sqrt{\tau}\rho_{01}+\theta_2\rho_{02})$, $a_1^4=a_1^0+(\sigma_\delta\sqrt{\tau}+\theta_2\rho_{12})$.

本文研究结果可以为将来国内市场出现该期权时提供一个较为理性的定价指导.在本文研究的基础上,还可以进一步考虑国内外无风险利率参数 r_{f} 和 r_{d} 不是常数而是一个随机过程的情况,那么在传统模型的基础上,假定利率遵从高斯利率过程、违约强度函数遵从重 Poisson 随机过程的情况下,同样可以给出不完全市场下[14-15] 该期权在双重风险暴露下的定价公式.

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