文章编号: 1004-4353(2014)02-0125-05

非简并光学参量振荡器中量子态频率上转换

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摘要: 提出一种在光学参量振荡腔中的量子态频率上转换的方案.方案分析了信号传递系数和转换效率随不同泵浦参数的变化情况,并定性和定量地阐述了频率上转换过程的量子性质.研究表明,泵浦参数在阈值以上并接近阈值时,量子态的转换频率可达到最大值.

关键词: 频率上转换; 传递系数; 转换效率
中图分类号: O431 **文献标识码:** A

Quantum state transfer in frequency conversion via nondegenerate optical parametric amplifier

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Abstract: This paper presents the scheme of the quantum state transfer via nondegenerate optical parametric oscillator. Analysis of the change of the transfer coefficient and conversion efficiency on the pump parameter, expatiation on the characterization of quantum frequency conversion process on the qualitative and quantitative. Frequency conversion efficiency of quantum states can be the maximum to closer to the threshold as long as the frequency conversion via operating optical parametric oscillator above the threshold.

Key words: frequency conversion; transfer coefficient; conversion efficiency

参量频率转换能够产生可调谐的相干辐射和压缩光[1],因此被广泛应用于干涉测量、精确测量和光谱学等方面,并成为量子网络的重要组成部分[2].在量子网络中,实现量子态的单位转换的方法有很多,例如可以通过粒子湮灭或产生来实现,也可以通过参量频率上转换来实现,等等.由于参量频率上转换能够显著提高转换效率[3],因此被认为是目前解决量子态转换的最佳方法,但在所知文献中其转换效率均低于 50%.研究[4-5]表明,在共振腔内进行频率转换可以有效增强转换效率.基于文献[5],本文提出了在光学参量振荡腔中的量子态频率上转换的方案.

1 理论模型

量子态频率上转换方案是利用内腔泵浦产生二次谐波的过程,如图 1 所示.系统由 3 个模式组成,其中包含一个谐波模 $\hat{a}_0$ 和两个分谐波模 $\hat{a}_1$, $\hat{a}_2$,其频率分别为 ω_0, ω_1 和 ω_2,且能量守恒满足 $\omega_0=\omega_1+$

收稿日期: 2013－10－22 **基金项目:** 延边大学自然科学基金资助项目(602013139)
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ω_2. 3个模式由光学腔内的二阶非线性晶体耦合在一起. 用非谐波场算符 $\hat{a}_1$,$\hat{a}_2$ 和谐波场算符 $\hat{a}_0$ 分别代表信号场、闲置场和谐波场. 低频量子态记作 $\hat{a}_1^{\text{in}}$(输入分谐波模),转换后的高频量子态记作 $\hat{a}_0^{\text{out}}$(输出谐波模),假设无噪声时转换效率可以达到100%,则

$$\delta^2 X(Y)_0^{\text{out}} = \delta^2 X(Y)_1^{\text{in}}, \tag{1}$$

$$\langle \hat{n}_0^{\text{out}} \rangle = \langle \hat{n}_1^{\text{in}} \rangle. \tag{2}$$

式中 $\delta^2 X(Y)$ 表示正交振幅(位相)组分的起伏,$\langle \hat{n} \rangle$ 表示对应模的平均光子数. 此时,频率转换从低频 ω_1 到高频 ω_0,用来衡量频率转换性能的信号传递效率 $T_{X(Y)}$ 和转换效率 η[6-7] 通常定义为:

$$T_{X(Y)} = \frac{SNR[X(Y)_0^{\text{out}}]}{SNR[X(Y)_1^{\text{in}}]}, \tag{3}$$

$$\eta = \frac{\langle \hat{n}_0^{\text{out}} \rangle}{\langle \hat{n}_1^{\text{in}} \rangle} = \frac{\langle (\hat{a}_0^{\text{out}})^+ \hat{a}_0^{\text{out}} \rangle}{\langle (\hat{a}_1^{\text{in}})^+ \hat{a}_1^{\text{in}} \rangle}, \tag{4}$$

其中 $SNR[X(Y)^{\text{in(out)}}]$ 表示输入(输出)模的正交振幅(位相)信噪比. 对于理想状态下的量子态频率转换,输出谐波模 $\hat{a}_0^{\text{out}}$ 的信噪比与输入分谐波模 $\hat{a}_1^{\text{in}}$ 的相同,即 $T_{X(Y)}=1$;两个模的平均光子数相等,即 $\eta=1$. 频率转换前后相空间中的正交组分保持不变.

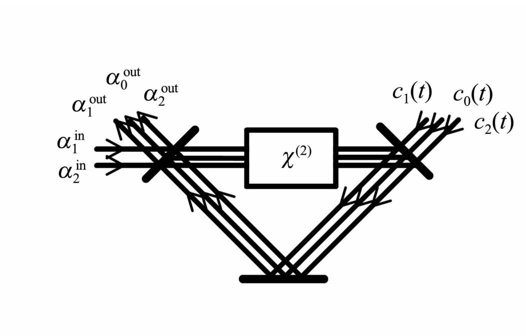

图1 实验装置图

在三共振光学腔的倍频过程中,两个分谐波模($\hat{a}_1$ 和 $\hat{a}_2$)与谐波模($\hat{a}_0$)在腔内同时共振. 在理想状态(相位高度匹配、零失谐和低损耗)下,输入和输出耦合系统的演化方程[8] 为:

$$\tau \dot{\hat{a}}_0(t) = -\gamma_0 \hat{a}_0(t) - \chi \hat{a}_1(t)\hat{a}_2(t) + \sqrt{2\gamma_0}\,\hat{c}_0(t), \tag{5}$$

$$\tau \dot{\hat{a}}_1(t) = -(\gamma_1+\mu_1)\hat{a}_1(t) + \chi \hat{a}_2^+(t)\hat{a}_0(t) + \sqrt{2\gamma_1}\,\hat{a}_1^{\text{in}}(t)\mathrm{e}^{\mathrm{i}\phi_1} + \sqrt{2\mu_1}\,\hat{c}_1(t), \tag{6}$$

$$\tau \dot{\hat{a}}_2(t) = -(\gamma_2+\mu_2)\hat{a}_2(t) + \chi \hat{a}_1^+(t)\hat{a}_0(t) + \sqrt{2\gamma_2}\,\hat{a}_2^{\text{in}}(t)\mathrm{e}^{\mathrm{i}\phi_2} + \sqrt{2\mu_2}\,\hat{c}_2(t), \tag{7}$$

其中:$\hat{a}_i^{\text{in}}(i=1,2)$ 表示输入振幅算符;$\hat{c}_j(t)(j=0,1,2)$ 表示真空噪声项对应的内腔损耗;χ 表示有效非线性耦合参数,与晶体二阶磁感应系数 $\chi^{(2)}$ 成比例. 假设信号场、闲置场和谐波场在光学参量振荡腔内循环一周的时间 τ 均相同,则总损耗参数是 $\gamma_i+\rho_i(i=0,1,2)$,其中 γ_i 与耦合镜振幅反射和透射系数相关,$\rho_i(i=0,1,2)$ 代表额外内腔损耗参数.

假设两个输入分谐波模 $\hat{a}_1^{\text{in}}$ 和 $\hat{a}_2^{\text{in}}$ 分别有实振幅 β_1 和 β_2,输入谐波模 $\hat{a}_0^{\text{in}}$ 为真空模,初相位满足 $\theta_{10}=\frac{\pi}{4}$ 和 $\theta_{00}=\theta_{20}=0$,两个分谐波模的腔透射因子和额外损耗均相同,即 $\gamma_1=\gamma_2=\gamma$,$\rho_1=\rho_2=\rho$,则方程(5)—(7)变为[9]:

$$-\gamma_0\bar{\alpha}_0\mathrm{e}^{\mathrm{i}\theta_0} - \chi\bar{\alpha}_1\bar{\alpha}_2\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)} = 0, \tag{8}$$

$$-(\gamma+\mu)\bar{\alpha}_1\mathrm{e}^{\mathrm{i}\theta_1} + \chi\bar{\alpha}_2^*\bar{\alpha}_0\mathrm{e}^{\mathrm{i}(\theta_0-\theta_2)} + \sqrt{2\gamma}\beta_1\mathrm{e}^{\frac{\mathrm{i}\pi}{4}} = 0, \tag{9}$$

$$-(\gamma+\mu)\bar{\alpha}_2\mathrm{e}^{\mathrm{i}\theta_2} + \chi\bar{\alpha}_1^*\bar{\alpha}_0\mathrm{e}^{\mathrm{i}(\theta_0-\theta_1)} + \sqrt{2\gamma}\beta_2 = 0, \tag{10}$$

其中 $\bar{\alpha}_0$，$\bar{\alpha}_1$ 和 $\bar{\alpha}_2$ 分别是内腔模 $\hat{a}_0$，$\hat{a}_1$ 和 $\hat{a}_2$ 的稳态解. 倍频振荡阈值 $\varepsilon_{th}=\sqrt{\dfrac{2(\gamma+\rho)^3(\gamma_0+\rho_3)}{\chi^2\gamma}}$，且 $\theta_1=\theta_0-\theta_2=\dfrac{\pi}{4}$. 由(8)—(10)式可得：

$$\bar{\alpha}_0=\frac{-\chi\bar{\alpha}_1\bar{\alpha}_2}{\gamma_0},\tag{11}$$

$$-(\gamma+\mu+\frac{\chi^2}{\gamma_0}|\bar{\alpha}_2|^2)\bar{\alpha}_1+\sqrt{2\gamma}\beta_1=0,\tag{12}$$

$$-(\gamma+\mu+\frac{\chi^2}{\gamma_0}|\bar{\alpha}_1|^2)\bar{\alpha}_2+\sqrt{2\gamma}\beta_2=0.\tag{13}$$

上述方程中的 $\bar{\alpha}_1$ 和 $\bar{\alpha}_2$ 是实数. 解方程(11)—(13)，可得阈值以上 3 个模的解为：

$$\bar{\alpha}_2=-\frac{\gamma+\mu}{\chi},\tag{14}$$

$$\bar{\alpha}_1=\frac{\sqrt{(\gamma+\mu)\gamma_0}\,\sigma}{\chi}-\frac{1}{\chi}\sqrt{(\gamma+\mu)(\sigma^2-1)\gamma_0},\tag{15}$$

$$\bar{\alpha}_2=\frac{\sqrt{(\gamma+\mu)\gamma_0}\,\sigma}{\chi}+\frac{1}{\chi}\sqrt{(\gamma+\mu)(\sigma^2-1)\gamma_0}.\tag{16}$$

根据输入输出关系[10] $\hat{a}_i^{in}+\hat{a}_i^{out}=\sqrt{2\gamma_i}\alpha_i$ 和阈值以上输出场振幅：

$$\alpha_0^{out}=-\frac{(\gamma+\mu)\sqrt{2\gamma_0}}{\chi},\tag{17}$$

$$\alpha_1^{out}=\sqrt{\frac{2(\gamma+\mu)\gamma_0}{\chi^2\gamma}}[\sigma\gamma-\sigma(\gamma+\mu)-\gamma\sqrt{\sigma^2-1}],\tag{18}$$

$$\alpha_2^{out}=\sqrt{\frac{2(\gamma+\mu)\gamma_0}{\chi^2\gamma}}[\sigma\gamma-\sigma(\gamma+\mu)+\gamma\sqrt{\sigma^2-1}],\tag{19}$$

可得转换效率 $\eta=\dfrac{|\sqrt{2\gamma_0}\bar{\alpha}_0|^2}{|\beta_1|^2}$. 将模式线性化为 $\hat{a}_i(t)=\bar{\alpha}_i+\delta\hat{a}_i(t)$，$\hat{a}_i^{in}(t)=\bar{\alpha}_i^{in}+\delta\hat{a}_i^{in}(t)$，$\hat{c}_i(t)=\delta\hat{c}_i(t)$ 后代入方程(5)—(7)，可得：

$$\tau\delta\dot{a}_0(t)=-(\gamma_0+\mu_0)\delta a_0(t)-\chi\bar{\alpha}_1(t)\delta\hat{a}_2(t)-\chi\bar{\alpha}_2(t)\delta a_1(t)+\sqrt{2\mu_0}\,\hat{c}_0(t),\tag{20}$$

$$\tau\delta\dot{a}_1(t)=-(\gamma+\mu)\delta a_1(t)+\chi\bar{\alpha}_2(t)\delta a_0(t)+\chi\bar{\alpha}_0(t)\delta a_2^+(t)+\sqrt{2\gamma}\,\delta\hat{b}_1(t)+\sqrt{2\mu}\,\hat{c}_1(t),\tag{21}$$

$$\tau\delta\dot{a}_2(t)=-(\gamma+\mu)\delta a_2(t)+\chi\bar{\alpha}_1(t)\delta a_0(t)+\chi\bar{\alpha}_0(t)\delta a_1^+(t)+\sqrt{2\gamma}\,\delta\hat{b}_2(t)+\sqrt{2\mu}\,\hat{c}_2(t).\tag{22}$$

用振幅和位相正交量定义 $X=a+a^+$ 和 $Y=-\mathrm{i}(a-a^+)$，获得输出场谐波模的起伏为：

$$\tau\delta\dot{X}_0(t)=-(\gamma_0+\mu_0)\delta X_0(t)-\chi\bar{\alpha}_1(t)\delta X_2(t)-\chi\bar{\alpha}_2(t)\delta X_1(t)+\sqrt{2\gamma_0}\,\delta X_{c0}(t),$$

$$\tau\delta\dot{X}_1(t)=-(\gamma+\mu)\delta X_1(t)+\chi\bar{\alpha}_2(t)\delta X_0(t)+\chi\bar{\alpha}_0(t)\delta X_2(t)+\sqrt{2\gamma}\,\delta X_{b1}(t)+\sqrt{2\mu}\,\delta X_{c2}(t),$$

$$\tau\delta\dot{X}_2(t)=-(\gamma+\mu)\delta X_2(t)+\chi\bar{\alpha}_1(t)\delta X_0(t)+\chi\bar{\alpha}_0(t)\delta X_1(t)+\sqrt{2\gamma}\,\delta X_{b2}(t)+\sqrt{2\mu}\,\delta X_{c2}(t),$$

$$\tau\delta\dot{Y}_0(t)=-(\gamma_0+\mu_0)\delta Y_0(t)-\chi\bar{\alpha}_1(t)\delta Y_2(t)-\chi\bar{\alpha}_2(t)\delta Y_1(t)+\sqrt{2\gamma_0}\,\delta Y_{c0}(t),$$

$$\tau\delta\dot{Y}_1(t)=-(\gamma+\mu)\delta Y_1(t)+\chi\bar{\alpha}_2(t)\delta Y_0(t)-\chi\bar{\alpha}_0(t)\delta Y_2(t)+\sqrt{2\gamma}\,\delta Y_{b1}(t)+\sqrt{2\mu}\,\delta Y_{c2}(t),$$

$$\tau\delta\dot{Y}_2(t)=-(\gamma+\mu)\delta Y_2(t)+\chi\bar{\alpha}_1(t)\delta Y_0(t)-\chi\bar{\alpha}_0(t)\delta Y_1(t)+\sqrt{2\gamma}\,\delta Y_{b2}(t)+\sqrt{2\mu}\,\delta Y_{c2}(t).$$

经过傅里叶变换后可得到：

$$\delta X_0(\Omega)=-1/\{\Omega^2(\mathrm{i}\Omega+\gamma_0+\mu_0)(\gamma+\mu)+2\Omega(\Omega-\mathrm{i}\mu_0-2\mathrm{i}\gamma_0\sigma^2)(\gamma+\mu)^2+$$

$$4\gamma_0(1-\sigma^2)(\gamma+\mu)^3[\mathrm{i}\sqrt{2}\Omega\gamma_0(\gamma+\mu)(2\gamma+2\mu+\mathrm{i}\Omega)\delta X_{c0}+$$

$$\sqrt{2(\gamma+\mu)\gamma_0}(\gamma+\mu)(-\mathrm{i}\Omega\sigma+(2\gamma+2\mu+\mathrm{i}\Omega)\sqrt{\sigma^2-1})(\sqrt{\gamma}\delta X_{b1}+\sqrt{\mu}\delta X_{c1})+$$
$$\sqrt{2(\gamma+\mu)\gamma_0}(\gamma+\mu)(-\mathrm{i}\Omega\sigma+(2\gamma+2\mu+\mathrm{i}\Omega)\sqrt{\sigma^2-1})(\sqrt{\gamma}\delta X_{b2}+\sqrt{\mu}\delta X_{c2})]\}, \quad (23)$$

$$\delta Y_0(\Omega)=1/\{-\Omega^2(\mathrm{i}\Omega+\gamma_0+\mu_0)(\gamma+\mu)+2\Omega(-\Omega+\mathrm{i}\mu_0+2\mathrm{i}\gamma_0\sigma^2)(\gamma+\mu)^2+$$
$$4\gamma_0\sigma^2(\gamma+\mu)^3[\mathrm{i}\sqrt{2\gamma_0}\Omega(\gamma+\mu)(2\gamma+2\mu+\mathrm{i}\Omega)\delta X_{c0}-$$
$$\sqrt{2(\gamma+\mu)\gamma_0}(\gamma+\mu)((2\gamma+2\mu+\mathrm{i}\Omega)\sigma+\mathrm{i}\Omega\sqrt{\sigma^2-1})(\sqrt{\gamma}\delta X_{b1}+\sqrt{\mu}\delta X_{c1})-$$
$$\sqrt{2(\gamma+\mu)\gamma_0}(\gamma+\mu)((2\gamma+2\mu+\mathrm{i}\Omega)\sigma+\mathrm{i}\Omega\sqrt{\sigma^2-1})(\sqrt{\gamma}\delta X_{b2}+\sqrt{\mu}\delta X_{c2}]\}. \quad (24)$$

根据振幅(位相)的输入输出关系 $\delta X(\delta Y)^{\text{out}}+\delta X(\delta Y)^{\text{in}}=\sqrt{2\gamma_i}\,\delta X(\delta Y)$,可得:

$$\delta X_0^{\text{out}}(\Omega)=\frac{1}{R_1}(A_1\delta X_{c0}+B_1\delta X_{b1}+C_1\delta X_{b1}+D_1\delta X_{b2}+E_1\delta X_{c2}), \quad (25)$$

$$\delta Y_0^{\text{out}}(\Omega)=\frac{1}{R_2}(A_1\delta Y_{c0}+B_2\delta Y_{b1}+C_2\delta Y_{b1}+D_2\delta Y_{b2}+E_2\delta Y_{c2}), \quad (26)$$

其中:

$$R_1=\Omega^2(\mathrm{i}\Omega+\gamma_0+\mu_0)(\gamma+\mu)+2\Omega(\Omega-\mathrm{i}\mu_0-2\mathrm{i}\gamma_0\sigma^2)(\gamma+\mu)^2+4\gamma_0(1-\sigma^2)(\gamma+\mu)^3,$$
$$A_1=-2\mathrm{i}\Omega\sqrt{\gamma_0}\gamma_0(\gamma+\mu)(2\gamma+2\mu+\mathrm{i}\Omega)-\Omega^2(\mathrm{i}\Omega+\gamma_0+\mu_0)(\gamma+\mu)-$$
$$2\Omega(\Omega-\mathrm{i}\mu_0-2\mathrm{i}\gamma_0\sigma^2)(\gamma+\mu)^2-4\gamma_0(1-\sigma^2)(\gamma+\mu)^3,$$
$$B_1=2\gamma_0\sqrt{\gamma(\gamma+\mu)}(\gamma+\mu)[\mathrm{i}\Omega\sigma-(2\gamma+2\mu+\mathrm{i}\Omega)\sqrt{\sigma^2-1}],$$
$$C_1=2\gamma_0\sqrt{\mu(\gamma+\mu)}(\gamma+\mu)[\mathrm{i}\Omega\sigma-(2\gamma+2\mu+\mathrm{i}\Omega)\sqrt{\sigma^2-1}],$$
$$D_1=2\gamma_0\sqrt{\gamma(\gamma+\mu)}(\gamma+\mu)[\mathrm{i}\Omega\sigma-(2\gamma+2\mu+\mathrm{i}\Omega)\sqrt{\sigma^2-1}],$$
$$E_1=2\gamma_0\sqrt{\mu(\gamma+\mu)}(\gamma+\mu)[\mathrm{i}\Omega\sigma-(2\gamma+2\mu+\mathrm{i}\Omega)\sqrt{\sigma^2-1}].$$
$$R_2=-\Omega^2(\mathrm{i}\Omega+\gamma_0+\mu_0)(\gamma+\mu)+2\Omega(-\Omega+\mathrm{i}\mu_0+2\mathrm{i}\gamma_0\sigma^2)(\gamma+\mu)^2+4\gamma_0\sigma^2(\gamma+\mu)^3,$$
$$A_2=\mathrm{i}\sqrt{2\gamma_0}\Omega(\gamma+\mu)(2\gamma+2\mu+\mathrm{i}\Omega)+\Omega^2(\mathrm{i}\Omega+\gamma_0+\mu_0)(\gamma+\mu)-$$
$$2\Omega(-\Omega+\mathrm{i}\mu_0+2\mathrm{i}\gamma_0\sigma^2)(\gamma+\mu)^2-4\gamma_0\sigma^2(\gamma+\mu)^3,$$
$$B_2=-\sqrt{2(\gamma+\mu)\gamma\gamma_0}(\gamma+\mu)[(2\gamma+2\mu+\mathrm{i}\Omega)\sigma+\mathrm{i}\Omega\sqrt{\sigma^2-1}],$$
$$C_2=-\sqrt{2(\gamma+\mu)\mu\gamma_0}(\gamma+\mu)[(2\gamma+2\mu+\mathrm{i}\Omega)\sigma+\mathrm{i}\Omega\sqrt{\sigma^2-1}],$$
$$D_2=-\sqrt{2(\gamma+\mu)\gamma\gamma_0}(\gamma+\mu)[(2\gamma+2\mu+\mathrm{i}\Omega)\sigma+\mathrm{i}\Omega\sqrt{\sigma^2-1}],$$
$$E_2=-\sqrt{2(\gamma+\mu)\mu\gamma_0}(\gamma+\mu)[(2\gamma+2\mu+\mathrm{i}\Omega)\sigma+\mathrm{i}\Omega\sqrt{\sigma^2-1}].$$

将 $\delta^2X_i^{\text{in}}=\delta^2Y_i^{\text{in}}=1$ 代入方程(25)和(26)得:

$$T_X=\frac{SNR[X_0^{\text{out}}]}{SNR[X_1^{\text{in}}]}=\frac{A_1^2}{A_1^2+B_1^2+C_1^2+D_1^2+E_1^2}, \quad (27)$$

$$T_Y=\frac{SNR[Y_0^{\text{out}}]}{SNR[Y_1^{\text{in}}]}=\frac{A_2^2}{A_2^2+B_2^2+C_2^2+D_2^2+E_2^2}. \quad (28)$$

2 结果与讨论

信号传递效率 $T_{X(Y)}$ 和转换效率 η 随泵浦参数 σ 的变化如图 2 所示,其中 $\gamma=0.018$, $\mu=0.002$, $\gamma_0=0.001$. 当系统运行在阈值以上时:信号传递效率 $T_{X(Y)}$ 随着泵浦参数 σ 的增加而增加,其中 T_X 增长趋势较为明显,而 T_Y 相对迟缓;转换效率 η 随泵浦参数 σ 的增加而减小,当接近阈值时其转换效率达到最大,实现了从 $\hat{a}_1^{\text{in}}$ 和 $\hat{a}_2^{\text{in}}$ 到 $\hat{a}_0^{\text{out}}$ 的频率上转换. 以上分析表明,在光学参量振荡腔中,量子态频率上转换很容易被实现. 本方案可以通过成熟的倍频技术在实验上得以实现.

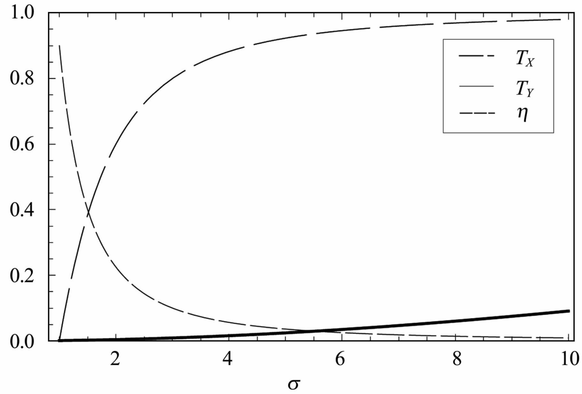

图 2　信号传递效率 T_X、T_Y 和转换效率 η 随泵浦参数 σ 的变化曲线

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